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Aspects of Word Frequencies

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by

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Preface

During the preparation, layouting and printing the book „Word frequency studies“ (2009)[1] a great number of new ideas about texts arose which could not be inserted any more in the above book. They appeared in form of articles dispersed in different journals and omnibus volumes and touched a very variegated palette of problems. We try to collect them and show the connections between them if there are any. Besides, we shall try to develop some of the ideas a step further. Frequently we shall take recourse to the above mentioned book whose knowledge is, however, not presupposed. If necessary, the pertinent object will be explained.

The booklet can be used as a collection of lectures in textology for a seminary and can be managed in one semester, even without a teacher. At the same time, the methods presented in both books can be used for text mining.

Since the individual chapters are heterogeneous developments of different issues, there is sometimes no logical nexus between the subsequent chapters. This is caused also by the fact that textology is no closed discipline and develops very quickly in different directions. It extends especially to the study of modern forms of texts, namely SMS, SPAM, E-mail and Internet pages, all of which display some divergent properties brought about by the conditions of the medium and the purpose. We restrict ourselves to literary texts but the methods can be applied to these special texts mutatis mutandis.

In many chapters we try to show the way from text to language typology, text being the surface where one can find the reflections of language structure. Needless to say, this is only the beginning of an enterprise which can be developed more extensively. Combining the properties and processes in the deep layers of language and on the surface represented by texts one will perhaps be able to construct some time a theory encompassing both. It will not have an algebraic structure, it will not concern grammatical rules and it will not be deterministic. It can turn out to become anything else but it will not be able to avoid probability, the basis of communication.

We want to express our gratitude to all those who took part in the sampling of texts for the above mentioned book and whose results are used in this book, too, namely P. Grzybek, B.D. Jayaram, R. Köhler, V. Krupa, R. Pustet, L. Uhlířová, and M.N. Vidya.

In the first place we want to thank Fengxiang Fan for his thorough reading of the book and correcting our Middle-European English. All remaining errors were made after his reading the book.

I.-I.P., J.M., G.A.

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[1] Popescu, I.-I., Altmann, G., Grzybek, P., Jayaram, B.D., Köhler, R., Krupa, V., Mačutek, J., Pustet, R., Uhlířová, L., Vidya, M.N. (2009). *Word frequency studies*. Berlin/New York: Mouton de Gruyter.

Contents

1. On text theory

The concept “text theory“ sounds similar to the famous concept “theory of everything”. Though in physics ever more disciplines have been brought under the same roof, and the same tendency can be seen in mathematics (sets, categories, measure theory), in textology a diverging tendency can be observed. Not only because the concept of text itself diverges widely since the existence of the Internet – there are not only sentences but also figures, lists, tables, navigation buttons, templates, etc. – but even without Internet the text, whether spoken or written, is a system consisting of physical, biological, linguistic, stylistic, psychological, social, aesthetic, emotional, attitudinal, dialectal, idiolectal, valuational, metaphoric, reverential, speech act, etc. entities whose investigation resulted in the rise of many different disciplines, and their number is still increasing. From time to time one takes a result from one of the disciplines and uses it as a constant parameter in another, but a “grand unification” has not even been considered as a research problem. Text is a kind of a multidimensional world whose easiest entrance is its outer form, namely the material sequence of linguistic entities. Both the entities and the relations between them are historically stabilized conventions even if some of them still have an iconic character. A written text has at least a fixed form but a spoken text is each time different.

If a researcher restricts himself to one of the above aspects, say, linguistics, he is in turn confronted with an overwhelming number of levels, units, properties, interrelations, aspects, impacts of other disciplines and so on. Problems should be solved, even partially, empirical hypotheses should be set up, in better cases a hypothesis should be derived from assumptions which play the role of preliminary axioms, statistical tests should be performed on the hypothesis and then the result interpreted. This is the normal way of any empirical science which passed the level of a proto-science. There are many disciplines in linguistics which content themselves with descriptions and classifications, avoiding any kind of test and hypotheses formation. Nevertheless, these disciplines are useful for practical purposes, e.g. the normalizing of the standard language, language learning, and their role must not be regarded lightly because they provide the basis for any deeper investigation.

Though it is preliminarily not possible to establish a general text theory, it is perhaps possible to investigate individual aspects, study the behaviour of selected entities and establish at least partial theories (called sometimes “teoritas”). To this end one needs concepts, conventions and hypotheses.

The *concepts* concern things, properties, relations, structures, functions, processes, history and systems. Peculiar enough, the concept of text itself is not firmly established and the proposed definitions remind us of the dozens of definitions of word and sentence. In the end, all of the definitions would boil down to the tautology “text is what we define as text”. Any other definition is either non-operational or refers to further undefined concepts. Text is not necessarily a

sequential entity (cf. Internet), it does not consist necessarily of linguistic entities (e.g. figures, tables, formulas, navigation buttons); on the other hand, even the life of an individual, the history of mankind etc. can be considered as texts because they share the time axis with texts. Not to mention musical texts, which have a number of common properties with language texts but may consist of more than one simultaneous time-dependent sequences. Thus linguistic text is a special case of a superior system which has not even been captured conceptually as yet. Operationally, we consider it a continuous linguistic sequence having a beginning and an end and having a hierarchic organization of levels and units abiding by Menzerath´s law. In the present volume we shall consider only texts of this kind, i.e. texts in the classical sense, excluding lists, Internet pages etc.

However, even this operationalization must be specified. First, we consider only written texts with an objective form and different from their interpretation or comprehension, which may form the external properties of texts and which have rather a subjective character (cf. Hřebíček 1997). Second, we consider a very restricted aspect, namely the word-form frequency and try to capture its properties and behaviour in order to characterize texts and languages and to propose law candidates.

As to *properties*, it is popular to consider them as something dwelling in objects, their intrinsic quality, but closer examination shows that this view is not quite adequate. The famous sentence “grass is green” means that grass “has” the green colour, but a physicist can easily show that green is rather a property of light; a physiologist can show that it is rather a property of our perception organs and daltonists and entomologists would agree. Last but not least, a linguist could say that “green” is merely a property of language because there are languages in which there is no such word and there are languages whose “green” is simultaneously our “green” and our “blue” (e.g. Japanese *aoi*), and there are languages in which there are different kinds of “green”. A methodologist adheres rather to the persuasion that things necessarily exist and have some qualities, but the “properties” we ascribe to them are our conceptual constructs. In advanced sciences these constructs are quantitative because in that form they are more precise. If we measure for example the length of an object, we do not ascribe values to the object but to our concept of length applied to the given object. Objects may exist in dimensions but they do not “have length”. Length is our concept. The pre-scientific Man uses qualitative concepts, which are sufficient for orientation and survival, but in science one uses quantitative concepts whose advantages are well known. Qualities and quantities do not exist in reality; they are properties of our concepts. They do not depend of the nature of objects but on the advancement of science, i.e. they are no “reflections” of reality.

Starting from these assumptions we see that

(1) language and its entities have potentially an infinite number of properties. Their increase proceeds automatically with the advancement of science in which one necessarily devises new concepts. The properties can be quan-

tified or operationalized in different ways; there is no "true" operationalization. There are no "natural" properties in language.

(2) All properties are measurable. Measurement units are conventions, and scales are numerical systems in which some operations are allowed.

(3) No property can attain an infinite value but it can be missing in language. Thus quantitative indicators of properties should vary in an interval whose right boundary is finite.

(4) There are no isolated properties, i.e. every property is linked with at least one other property. The net of these linkages gives rise to structure. At the same time, a set of linkages is controlled by self-regulation without which any communication would be destroyed. In other words, for any set of linkages there is an attractor caring for communicative equilibrium.

(5) Every property changes, or better, the degree of every property changes. But if a property changes, all other properties linked with it must change, too, otherwise the self-regulation caring for effective communication would be destroyed.

(6) All properties of language realized in texts are random variables following a "proper" distribution. The same holds for the frequencies of members of different classes which follow a regular rank-frequency distribution or form a regular rank-frequency sequence.

(7) Since a "classical" text is a linear formation, each property forms some linear patterns which can be modelled.

(8) Every property contributes or gives rise to a special quality of text which is eo ipso measurable. Thus the Galilean requirement holds for the textology, too (for more detail see Hřebíček, Altmann 1996; Altmann 2001, 2006.)

Conventions, such as definitions, operations, rules, symbols, criteria are necessary components of science but they do not have any truth value. However, definitions are a frequent stumbling-block of analysis. For some concepts there are dozens of definitions whose authors tried to capture the (not existing) "essence" of a linguistic entity. Of course, definitions are necessary because we must know what we speak about, but for text analysis we need operational definitions which allow us to identify the entities and to partition the text. Even the operational definitions of the same entity may be different but none of them is more true than the other ones. Operational definitions are based on criteria which are not present in data but are conceptually constructed by us. Consider for example the word-form. In every language there are several segmentation possibilities. One must decide(sic!) whether hyphenated words represent one or two word-forms; numbers like 2128 can be considered one word-form or as many words as there are digits in the number (or even more); in Slavic languages there are zero-syllabic prepositions behaving phonetically like proclitics but morphologically as independent words; in Slovak, syllabic prepositions (often) take over the main accent of the word but linguists ignore this phonetic rule in their decisions about word-form, though the same prepositions are written together

with verbs and nouns as prefixes, etc. Now, whatever our decision, operational definitions are either prolific or not prolific. They are prolific if the result of the analysis is in agreement with an a priori hypothesis. On the other hand, a hypothesis may hold true only if the text is analyzed in a given way. Thus a hypothesis may represent an external criterion deciding about the best way of analyzing texts or languages. Nevertheless, there are still other external criteria, such as economy of inventory, symmetry of the system, etc. This boils down to the fact that hypotheses and operational definitions must be formulated hand in hand but sometimes different analyses may corroborate the same hypothesis. In any case, that operational definition is more prolific which better corroborates some a priori hypothesis.

Every statement we pronounce is a *hypothesis*, but in science we prefer statements of a special kind (cf. Bunge 1967). A textual hypothesis must be well-formed, it must contain some information concerning texts, it must be in principle objectively testable and it should fit the bulk of knowledge. The testing of a hypothesis is a long procedure. First it must be "translated" into textological language containing operationalized concepts helping to get data, then in a statistical hypothesis whose testing yields numbers, and these numbers must be interpreted in terms of acceptation or rejection. All textological hypotheses are probabilistic. The acceptation of a textological hypothesis does not mean a proof but merely a corroboration, because outside of mathematics no statement can be definitively proved. Textological hypotheses can be local, e.g. concerning the development of a feature of English texts, or global, concerning all texts in all languages. "Good" hypotheses are derivable from other hypotheses, laws, theories, axioms or even from reasonable textological/linguistic assumptions based on some general issues such as requirements of speaker/hearer, forces, self-regulation, etc. It is not reasonable to set up textological/linguistic hypotheses based on analogies with physics because this technique sometimes evokes the erroneous idea that linguistics can be reduced to physics. Textological entities do not behave like physical entities even if both underlie the same flow of time.

Since all empirical sciences contain both inductive and deductive hypotheses, in textology one can strive for partial *inductive-deductive theories*. In order to achieve this aim, at least one of the hypotheses must be a law. According to M. Bunge (1967: 381) "A scientific hypothesis (a grounded and testable formula) is a *law* statement if and only if (i) it is *general* in some respect and to some extent; (ii) it has been empirically *confirmed* in some domain in a satisfactory way, and (iii) it belongs to a scientific *system*."

A textologist should not strive for laws and theories as they are known in physics or chemistry but adapt the above definition for his own purposes. The requirement of *generality* means that a hypothesis must hold for all languages but it is sufficient if special texts are concerned. Or one adds the ceteris paribus condition eliminating all other texts; or one incorporates parameters representing a given linguistic level or a particular text sort. Generality is a matter of degree. Satisfactory empirical *confirmation* cannot be achieved if only English is scruti-

nized. The more languages and the more texts were used for the testing, the better can be the confirmation. Empirical confirmation is a matter of degree, too. Textological hypotheses can be confirmed only statistically. There is no dichotomy between "accepting" or "rejecting" a textological hypothesis but always a probability of error of accepting a "false" or rejecting a "true" hypothesis. It is a decision based on probability, but this probability (e.g. the significance level) is not a natural constant, it is a convention. The problem of exceptions is irrelevant and can be solved by enriching the hypothesis by taking into account additional variables or reserving place (parameters) for specific circumstances. The *scientific system* to which a hypothesis should belong need not be a fully axiomatized theory; it is sufficient to have a scientific framework from which hypotheses can be derived. For many textological problems this role may be played by synergetic linguistics. Needless to say, textological laws will never have the same status as physical laws – any comparison is useless. However, even in physics, many hypotheses arose inductively, but in 400 years of research performed by thousand of scientists it was easier to incorporate them in some scientific systems than in textology which presently begins to develop.

It is nothing wrong in developing a discipline in an inductive way. All empirical sciences began in this way. But at times the necessity of systematizing the collected knowledge looms up and theories arise. And since theories must contain at least one law, a scientific system must be set up from which they can be deduced. In the course of development, ever more hypotheses will be systematized and even if there are several systems forming membra disiecta at the beginning, after some time they will be unified.

Wimmer et al. (2003) recommend the following procedure as one of the many possible ones:

1. Observing a "conspicuous phenomenon" in a text set up a low-level hypothesis, i.e., use empirical concepts such as "in this text", "phoneme /a/", "words with meaning X", "in English", etc. That means, devise concepts and register a phenomenon.
2. Generalize the statement omitting empirical concepts, broaden the range of the hypothesis (e.g. "for all texts…", "for all languages…"), insert hypothetical conditions enabling us to consider different texts, etc. That means, broaden the hypothesis in order to encompass hitherto not observed or in principle not observable phenomena (Bunge 1967: 223ff.). Call this statement G (symbolizing a general statement).
3. Test this hypothesis on further data (texts, languages) with respect to boundary conditions represented by language, language level, text sort, speaker, etc. If G does not get corroborated, modify it. Every hypothesis must be corrigible, otherwise it is a dogma. If it gets corroborated, then
4. set up other hypotheses about the genesis, behaviour, course, form etc. of the phenomenon and join them with G. There are several alternatives: (a) One can derive some other hypotheses from G or (b) one can derive G from them; (c) G turns out to be merely a boundary condition represented

by a constant in another hypothesis or (d) the other hypothesis turns out to be G´s boundary condition; (e) the central entity of G has further properties whose relationship with the observed "conspicuous phenomenon" is the subject of further hypotheses. Etc. In this way gradually a network of relationships, a *system* of statements arises. At a higher level, some of the hypotheses can be selected as starting points (axioms) for the other ones.

In step 4 one approaches deduction and finds statements from which G may result. At this step the necessity of mathematics will be evident. Though in quantitative linguistics one uses it from the beginning, i.e., already in the first step when one must measure properties or count frequencies, the construction of a theory requires a little more mathematics in order to deduce some lawlike hypotheses. Thus, it is better to start with quantified concepts, especially those concerning properties.

As was said at the beginning, text represents a multidimensional world with many independent entrances. It is much richer than any natural science because it contains a potentially infinite number of non-material dimensions. Nevertheless, the approaches and views can be divided in some comprehensive classes.

Each of the analyses can be performed (i) globally, taking the whole text simultaneously and computing e.g. frequencies of an entity, or (ii) sequentially, observing the stepwise appearance of entities and studying sequential regularities.

One can study (a) one selected text, (b) several texts in the same language or (c) several texts in different languages. Approach (a) is punctual and yields information only about the given text or in psychiatry about the given patient. Approach (b) can yield information about the author, genre, language, "-lect" (dialect, idiolect, sociolect) and the history/development of an entity. Approach (c) yields information about the variation of a property, its empirical limits, and opens a door to language typology.

The aims of the analysis may be (A) descriptive, classificatory (with at least approach (b)), comparative or historical. If performed with quantitative concepts, statistical methods are necessary. (B) Theoretical, studying the relationships between entities, setting up empirical hypotheses and testing them on individual texts. Here, individual texts are only testing instances. The relations between theoretical entities are derived deductively and give rise to lawlike statements. The best example is synergetic linguistics.

The combination of all these aspects yields a scientific discipline whose boundaries are not determinable. At present, in spite of the fact that some lawlike hypotheses have already been proposed and well corroborated, we stay at the entrance of an enormous research domain. Unfortunately, only a few textologists are ready to use quantitative methods, the great majority is not favourably inclined towards any kind of mathematics. The latter community is dominated by four kinds of error´s which can easily be classified as different *idola* of Francis Bacon (cf. Altmann 1999; Wimmer et al. 2003): (i) "Our objects cannot be

mathematized/quantified". As has been shown above, we perform operations only with our concepts, not with real objects. We ascribe degrees to our concepts of properties and associate them with objects. We do not mathematize reality but our ripe concepts of reality. (ii) "Even if it would be possible, we are interested in qualities, not in numbers". This is confusing ontology with epistemology. In order to recognize the reality we are forced to form concepts using the weak electric impulses penetrating through our receptors into the brain, i.e. we construct the reality. Conditioned ontogenetically we first form qualitative concepts which are partially embodied in and conveyed by our language, later on we learn the quantitative concepts which allow us to express everything more precisely. Thus the ontogenetic order of cognitional successes seduces us to believe that reality is qualitative. It must be remarked that no quantitative textologist has ever been interested in numbers but always in properties, relations, structures, processes and systems concealed behind the numbers. (iii) "We are not interested in text laws but in the uniqueness, idiosyncrasy of texts." But uniqueness or idiosyncrasy can be stated only as a contrast to something else or seen on a general common background formed by a theory which may be quite primitive at the beginning. The aims of science are not (only) descriptions but above all theories consisting of general, testable statements. Idiosyncrasies are *cura posterior* even if empirical research begins with them. In the framework of a theory they are deviations caused by initial, boundary or supplementary etc. conditions. (iv) "Our problems are that complex that no mathematics can capture them". Both verbal and mathematical models are simplifications. Usually some aspects are analyzed in isolation, different relations are omitted and left constant (*ceteris paribus*). It is not clear, how the complexity of phenomena could be described more exactly by a natural language which is full of fuzziness, inexactness, ambiguity etc. The collecting of descriptive data made by means of natural language is a prerequisite for an analysis but not the analysis itself.

For textologists it is not easy to give up even one of these *idola* as long as the teaching at universities moves within the comfortable world whose boundaries are fixed and seem to be incontestable. Theory construction does not mean a destruction of this hermetic world; it only means the opening of several windows in the same way as it has been done in many other sciences.

2. On sampling and homogeneity

The preparation of a text for any kind of analysis is a very complex act. An "all-purpose" preparation is impossible because one will never know all possible purposes, and the preparation for a special purpose always contains ambiguities which would be solved differently by different linguists, and this would lead to lasting controversies. In many cases subjective and authoritative decisions are necessary.

But let us assume that such a preparation took place. For grammatical purposes the sample should be as large as possible, i.e. the ideal data is a contemporary corpus in which at least the identity of the given language must be warranted (e.g., which English is "the English"? What does "contemporary" mean? What does "text type" mean? etc.) The sample´s linguistic homogeneity should be given as defined. However, for many other scientific purposes texts conceal two kinds of inhomogeneity:

(1) Long texts are automatically inhomogeneous because they cannot be written in one go. If the writer makes pauses (for sleeping, eating, coffee, etc.), some rhythms change in his brain and the continuation of the text may display some changed properties, e.g. change in sentence length and structure, words, sentiments, etc. In spite of this fact, some regularities remain untouched, or the change is so small that only very sophisticated statistical techniques could detect it. On the other hand, the statistical dictum "the larger the sample, the more reliable are the results" does not hold in textology (but perhaps in grammar), but the motto "the larger the sample, the more inhomogeneous is the text" does. The classical statistical tests usually fail when applied to corpora because the smallest difference in proportions can be made significant if a sufficiently large sample size is taken. That is to say, some classical tests are not reliable any more. Surely, some properties get stabilized with increasing text length, e.g. letter frequencies, other ones may display chaotic behaviour because of text mixing. Unfortunately, the study of text homogeneity or the steady state of properties in texts has been touched very sporadically up to now (cf. Hřebíček 2000). Some researchers believe that Zipf-Mandelbrot´s law holds for whole texts, others believe that it holds for parts, too. The emphasis is on "believe".

(2) In a certain sense, living systems can be classified in homogeneous and non-homogeneous ones. Non-homogeneous systems are those whose parts (components, elements, organs) are different, e.g. organisms, but at the microscopic level they may be homogeneous, consisting of cells. A text can be considered homogeneous if clauses, words or syllables are considered its elements. For homogeneous systems Menzerath´s law holds; for non-homogeneous systems its counterpart, the allometric law holds. Both are power functions but the exponents have different signs. But if, at a certain linguistic level, we do not consider the entities as a uniform class, e.g. the class of "words" is partitioned in parts of speech, the text automatically gets non-homogeneous; its entities abide

by different regularities influenced by those of other entities, there are dependencies, interrelations, etc.

Let us illustrate these circumstances by two examples. Consider the word length distribution in Goethe's Letters. Adhering to Grotjahn´s proposal (1982), who for this purpose randomized the parameter of the displaced Poisson distribution by the gamma distribution and obtained the displaced negative binomial distribution[1]. We fit the latter to two Letters of Goethe. Taking the texts separately, it yields an excellent fit, as shown in Table 2.1. But if we add the data of the two Letters, we still obtain the negative binomial; however, the fit is poorer. The parameters *k* of the two letters are too different to belong to the same population. A chi-square test for homogeneity corroborates it. However, this may simply be caused by the very disagreeable property of the chi-square, which increases with increasing sample size.

Table 2.1
Fitting the 1-displaced negative binomial distribution to Goethe´s Letters[2]
(Altmann 1992)

	Letter No. 612		Letter No. 647		612 + 647	
x	f_x	NP_x	f_x	NP_x	f_x	NP_x
1	164	162.68	259	259.16	423	422.36
2	105	104.13	132	125.65	237	230.40
3	35	38.83	37	46.65	72	84.96
4	15	11.02	19	15.55	34	26.32
5	1	3.33	6	4.89	7	7.38
6	-	-	1	2.10	1	2.58
	k = 6.0542 p = 0.8942 X^2 = 3.46 DF = 2 P = 0.18		k = 2.0100 p = 0.7545 X^2 = 3.95 DF = 3 P = 0.27		k = 3.1764 p = 0.8246 X^2 = 5.51 DF = 3 P = 0.14	

Thus all Letters of Goethe, if added, do not constitute a homogeneous population, and if we add more of his works, we do not obtain a word length population called "Goethe". If we add works by other writers, we do not set up a word length (or other) population called "German" – a fact discovered long ago by J.K. Orlov (1982). Thus a word length population could be truly represented by a

[1] A number of other distributions capturing word length can be found in Wimmer, Altmann (1996), Wimmer, Witkovský, Altmann (1999).
[2] The slight differences in results are due to the improvement of the software.

weighted sum of probability mass functions. If one would consider also the part-of-speech character of words, one would obtain a sum of multidimensional probability distributions. Needless to say, every science is simplification and approximation, and we are not always aware or not forced to be aware of the basic non-homogeneity of our data.

The next case shows that one can avoid non-homogeneity simply by leaving a certain property aside. A typical case is that of rank-frequency distribution of words. One sets up this distribution considering all words as equivalent and ignores the fact that they belong to different parts-of-speech (or other) classes each having its own distribution. Consider an artificial example shown by Popescu, Altmann, Köhler (2009) and presented in Table 2.2. We divide the words in 4 classes and for each class we obtain the rank-frequencies. In a hypothetical case we may obtain the distributions in Table 2.2. Here in most of the cases the same rank consists of quite different frequencies, and the theoretical distributions – even if they would be of the same type – would have different values of parameters. The situation is graphically presented in Figure 2.1.

Table 2.2
Ranking of different components

rank	f_1	f_2	f_3	f_4
1	30	16	8	4
2	20	12	5	2
3	12	8	2	1
4	7	5	1	1
5	4	2	1	1
6	2	1	1	
7	1	1		
8	1	1		
9	1	1		
10	1	1		

Now, if we ignore the classes, i.e., homogenize the result, then we re-rank the whole field in such a way that the higher frequency of any element attains automatically a lower rank and the lower frequencies of all components are placed at the next ranks. In this way, we obtain the usual picture of rank-frequency distributions. In the above example we obtain the overall frequency sequence: 30, 20, 16, 12, 12, 8, 8, 7, 5, 5, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1. Since it is simpler to consider one global case than many local ones, the first approximation yielding a more realistic image would not be the recourse to the simple Zipf-distribution or its generalizations but rather to a superposition of functions

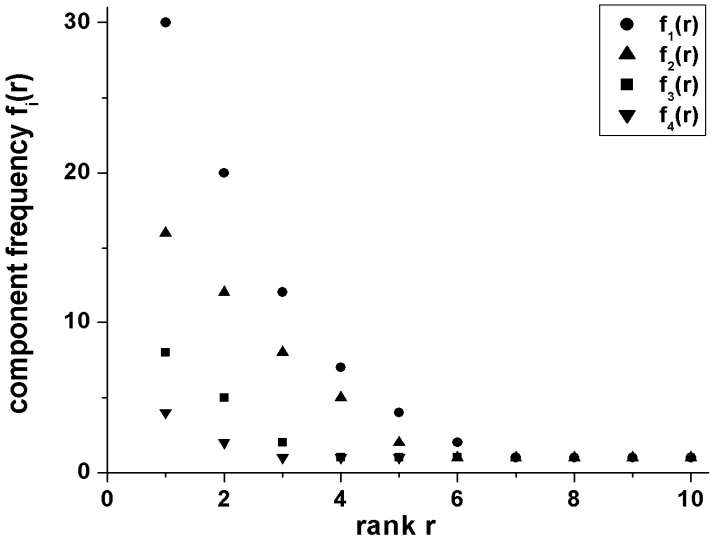


Figure 2.1. Rank-frequency distributions with partition of the overall class into four components

capturing each stratum separately and summing up to a common function. As a matter of fact, this is possible, at least in this case, as will be shown in Chapter 3. Such a homogenization of the data from Table 2.2 is shown in Figure 2.2. The strata disappear but can be reconstructed in the model.

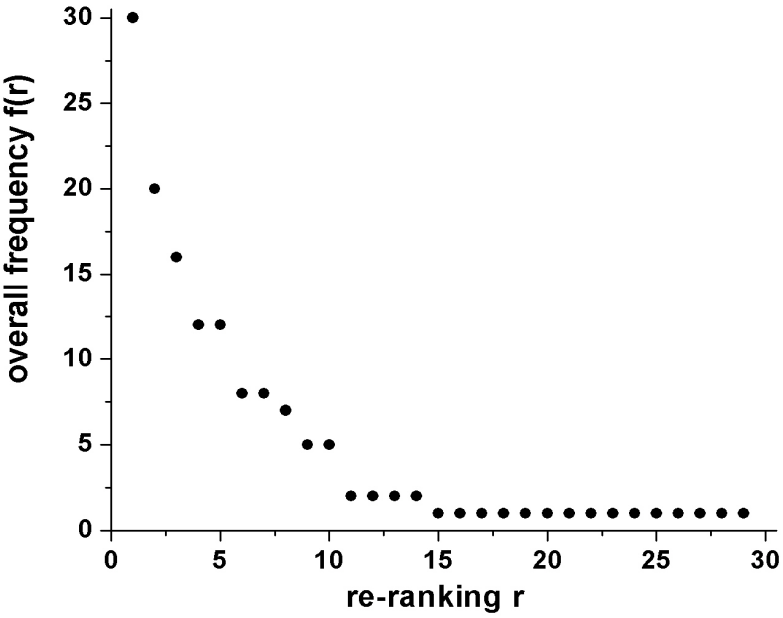


Figure 2.2. Re-ranking (homogenizing) the data in Table 2.2

In spite of non-homogeneities, any textual phenomenon can be modelled, but we should not see in our models an “eternal truth”. They are simplifications, approximations, and conscious omissions of aspects which are not in the focus of our analysis. They are pieces of mosaic by means of which we try to compose a picture whose contents is unknown and can be re-built any time.

Thus, sampling in textology should be purposeful. We should not forget that data are not given a priori and we do not collect them like strawberries but we create them. They are conceptual creations in the same sense as the concepts themselves that refer to them. They should be based on a hypothesis and collected using operationalized criteria which are not contained in the data but are again our concepts created for the given purpose. In textology, sometimes the authoritative sampling may be more adequate than random or systematic sampling.

3. A new view on Zipf´s law

"Zipf´s law" is the general name by which the relation between the frequency f_r and the rank r of any linguistic elements has been baptized. The number of formulas capturing this regularity is enormous (cf. http://www.nslij-genetics.org/wli/zipf/ created by Wentian Li). Their exuberance was caused by two circumstances: (1) The researchers tried to give it a better substantiation than Zipf and arrived automatically at different formulas. A part of the formulas are generalizations of the zeta distribution, another part are empirical modifications of Mandelbrot´s approach (which is itself a generalization of zeta) and a third part contains ad hoc trials based sometimes on analogies, proliferates in serendipity and is full of endeavours to obtain a better fit to data. Since the new mathematical trials are made mostly by non-linguists, a failure in fitting a model to the given data is in turn corrected by a modification of the model. This technique enhances mathematical complexity. (2) However, in case of a failure, linguists concentrate on the first of the three steps of improvement: (a) check the data, (b) check the computation, (c) check the model. Since computations are performed today by means of a computer, textologists may concentrate only on data. But even textologists believe that data are given a priori. However, before one collects data, one must create them in the light of a hypothesis. A very remarkable example is, e.g., the modelling of the distribution of sentence length in texts having no punctuation e.g. Early High German official records. Here, "sentence length" is a concept which must be operationalized very exactly, otherwise one document = one sentence, then data are created from the text and, in the end, sentence length can be measured. In simple cases textologists operationalize a property and the measurement unit, but the non-homogeneity of the text − mentioned in the previous chapter − is mostly not worth of attention. In a stage play there are as many parts as there are acts and as many strata as there are actors. If we mix up the shares of all actors in a unique data file, we sometimes obtain a not very regular function which significantly deviates from our previous models. And though the non-homogeneity is not so conspicuous in many texts, we have, nevertheless, several different intrinsic and extrinsic non-homogeneities which must be taken into account. They are due to (objective and subjective) conditions under which the text was created.

In the case of word-form frequencies we have different parts of speech which can in turn be classified as autosemantics and synsemantics; there is the speech of the author and that of acting persons, the functions of words in the sentence, the restrictions of the genre, the technical domain within a genre, the style, etc. This all forms strata within which there is a leading element and a regularly decreasing share of the other elements. Here we conjecture further that within each stratum – if it is determined adequately – the decrease of frequencies of individual elements is very regular and has an exponential form or, considered in discrete steps, a form of a geometric sequence. If this conjecture is correct,

then it can serve as an external criterion for class forming in text or in language. If we ignore the strata and re-rank the frequencies according to their magnitude in the whole text, we obtain a superposition[1] of exponential elements which can be expressed as

$$(3.1) \quad f(r) = \sum_{i=1}^{n} A_i e^{-r/a_i} = A_1 e^{-r/a_1} + A_2 e^{-r/a_2} + \ldots + A_n e^{-r/a_n},$$

where n is the number of relevant strata, A_i is the amplitude, a_i the decay coefficient, r is the rank and $f(r)$ is the frequency at rank r. Since the frequency cannot be smaller than 1, the formula can be improved by adding 1 to the sum, i.e. (Popescu, Altmann, Köhler 2009)

$$(3.2) \quad f(r) = 1 + \sum_{i=1}^{n} A_i e^{-r/a_i} = 1 + A_1 e^{-r/a_1} + A_2 e^{-r/a_2} + \ldots + A_n e^{-r/a_n},$$

which can be written also as

$$(3.3) \quad f(r) = 1 + \sum_{i=1}^{n} A_i q_i^r \ .$$

The sum (3.3) is a linear combination of the terms which are solutions of difference equations

$$\frac{\Delta P_{x-1}}{P_{x-1}} = \frac{P_x - P_{x-1}}{P_{x-1}} = a_i, \ i = 0,1,\ldots,n,$$

or, written in the equivalent form,

$$P_x = (1 + a_i) P_{x-1}, \ i = 0,1,\ldots,n.$$

It is a special case of the general approach introduced by Wimmer and Altmann (2005). The number of summands (n) is the greater, the more independent strata are in the text. However, with word-form frequencies usually not more than two strata are active, namely that of autosemantics and that of synsemantics which are not independent. If there is a well balanced cooperation, even one component of (3.2) is sufficient for capturing the data. If two components are necessary, then the strata can e.g. be stylistically differently emphasized. For example in a very

[1] The idea of modelling rank-frequency distributions with the aid of superpositions has a longer history, cf. e.g. Altmann (1992): "…if one does not separate the individual strata, the models of rank-frequency distributions should always be presented as superpositions of distributions."

ornamental style the tail of the sequence may be considerably prolonged. In that case one can identify the following components: that with great A_i but small q_i belonging to the quickly decreasing class, namely that of synsemantics, and that with small A_i but great q_i being the slowly decreasing class of autosemantics. The necessity of a third or of more components gives a possibility of philological interpretations.

Usually, one models the rank-frequency phenomenon using a discrete probability distribution. Zipf himself did not do it, he simply found a relationship, but was criticized for this reason (cf. e.g. Joos 1936), other researchers did it and were criticized, too, because rank is no random variable but a position in a sequence. This is probably the source of the myth of tautology of Zipf´s law. However, an ordered set is no myth and if the ordering can be expressed formally, there is neither an empirical nor a theoretical reason not to do it. Ranked frequencies simply represent a decreasing sequence of numbers, and the ranks are no measurable properties which could be ascribed to the words in the text by (operational) definition. If we do not expect more, there is no obstacle for seeking the form of this regularity.

What more, the theoretical sequence (3.2) can even be replaced by a continuous function which would be a different approximation to the ordering regularity. As already mentioned, continuity and discreteness are properties of our concepts used especially in mathematical models, and not properties of reality.

Needless to say, the model of the sequence can be normalized and in that case it yields a discrete probability distribution. The ranks play the role of an auxiliary random variable and no additional linguistic interpretation is necessary. The distributional approach has, however, some disadvantages. First, there are no homogeneous texts with infinite inventory, hence each theoretical distribution must be truncated at the right side. The majority of the models do not care for this circumstance. Second, the goodness-of-fit is estimated using the chi-square test whose weaknesses are well known. The major ones are: (i) the deviations in classes with great frequencies obtain a small weight but those with small frequencies great weights even if the small frequency can be caused by small sample size. That means, the goodness of fit can be decided on the basis of unreliable classes. Pooling of frequency classes helps to moderate this flaw. (ii) The weighting of deviations is asymmetric. The same difference between theoretical and empirical frequency can obtain different weight: if e.g. O = 5 and E = 3, the difference is |O-E| = 2, and the component of the chi-square yields $(5-3)^2/3 = 1.33$, but if O = 3 and E = 5, the component is $(3-5)^2/5 = 0.8$. (iii) The usual chi-square test is a special case of the Cressie-Read statistics, in which the exponent need not be 2 but can be optimized. (iv) The chi-square is a sum of squared normal variables, i.e. based on normality of deviations. But in language there are no normal deviations; all of them are made in favour of the speaker/writer; they are always skewed. In language there are not even normal distributions. Such a state, i.e. normality, contradicts the nature and the development of language, as

has been shown in many publications. (v) Many linguists consider the significance level, e.g. α = 0.05, as a mysterious holy entity existing somewhere in the reality. But it is merely a modest help for our personal decisions about accepting or rejecting the result. (vi) However, possibly the greatest deficiency is the dependence of the chi-square on sample size and on degrees of freedom. Frequently no degrees of freedom remain because of the necessity of pooling some classes in small samples. But many linguistic samples are enormously large and the chi-square grows arithmetically, i.e. it does not help us to decide if we fixed the significance level a priori. In order to alleviate the decision, different contingency coefficients (Cramér, Tschuproff, Pearson) have been proposed in which the sample size or even the degrees of freedom disappear but at the same time the significance level gets senseless. We must decide(!) which result will be accepted and which not.

Thus, using a sequence, we get rid of the chi-square and can carry out our decisions using the simple determination coefficient measuring the relative amount of "unexplained" deviations of data from the model.

In what follows, we show different cases from different languages. The "goodness-of-fit" or rather, the adequateness of (3.2) will be signalized by means of the determination coefficient

$$(3.4) \quad R^2 = 1 - \frac{\sum_{r=1}^{V} [f(r) - \hat{f}(r)]^2}{\sum_{r=1}^{V} [f(r) - \bar{f}]^2} ,$$

where $\hat{f}(r)$ are the values computed according to (3.2) and $\bar{f}$ is the mean of empirical frequencies. The numerator contains the squares of unexplained deviations and the denominator the total variation of frequencies. The distribution of this indicator is not known, an $R^2 > 0.9$ is considered a very good fit.

The fitting has been performed iteratively. In Table 3.1 the fitting of (3.2) to rank-frequency sequences in 100 texts in 20 languages is presented. The results are taken from Popescu, Altmann, Köhler (2009). In all cases two components of (3.2) were sufficient to capture the decrease of frequencies, as illustrated in Figure 3.1 for Goethe's Erlkoenig (G 17). The titles of texts are given in the Appendix of the book Popescu et al. „Word frequency studies" (2009).

Table 3.1
Fitting (3.2) to word rank-frequencies in 100 texts from 20 languages
(data taken from Popescu, Altmann, Köhler 2009)

(B = Bulgarian, Cz = Czech, E = English, G = German, H = Hungarian, Hw = Hawaiian, I = Italian, In = Indonesian, Kn = Kannada, Lk = Lakota, Lt = Latin, M = Maori, Mq = Marquesan, Mr = Marathi, R = Romanian, Rt = Rarotongan, Ru = Russian, Sl = Slovenian, Sm = Samoan, T = Tagalog)

ID	A_1	a_1	A_2	a_2	R^2 present	R^2 Zipf
B 01	47.6626	1.8310	11.7204	24.6430	0.9878	0.9837
B 02	11.8851	9.8591	1.8616	22.8208	0.9858	0.8705
B 03	11.3883	10.6940	3.8979	30.8570	0.9905	0.8790
B 04	18.2279	2.0296	9.5736	17.9197	0.9901	0.9619
B 05	12.2884	12.7005	10.4218	1.9141	0.9888	0.9367
Cz 01	89.8092	1.5034	10.0834	29.8773	0.9918	0.9764
Cz 02	139.1469	0.7742	17.2807	19.6472	0.9821	0.9767
Cz 03	425.0694	0.8345	54.2745	20.1402	0.9845	0.9832
Cz 04	70.6745	0.7588	7.1370	23.8965	0.9744	0.9537
Cz 05	194.2721	0.9509	15.1312	20.7528	0.9919	0.9715
E 01	142.7565	3.7490	16.3789	53.1611	0.9956	0.9620
E 02	166.6249	2.3215	47.3363	30.8223	0.9819	0.9661
E 03	262.9265	3.0351	34.0817	37.5747	0.9873	0.9752
E 04	507.9130	2.2501	39.3783	49.7372	0.9881	0.9870
E 05	339.0010	2.4760	58.7252	33.4563	0.9802	0.9822
E 07	255.4869	4.8007	35.4756	54.8410	0.9933	0.9347
E 13	793.2101	3.1952	106.0313	52.8682	0.9682	0.9800
G 05	32.3930	3.4098	4.6964	29.8436	0.9938	0.9646
G 09	25.7526	3.3468	8.1681	25.4876	0.9850	0.9626
G 10	17.3132	4.1391	4.6533	26.4403	0.9870	0.9402
G 11	17.1652	2.6471	5.8477	23.9797	0.9796	0.9593
G 12	25.1615	0.6162	8.9478	9.1158	0.9873	0.9514
G 14	7.0453	7.6272	5.3720	1.5982	0.9867	0.9349
G 17	6.3872	14.9185	6.1660	2.4572	0.9824	0.9349
H 01	692.3412	0.8109	21.2372	23.5043	0.9847	0.9600
H 02	1047.3379	0.4214	36.7323	6.3047	0.9809	0.9365
H 03	165.3035	0.7452	3.9648	14.8825	0.9962	0.8864
H 04	132.8165	1.5791	5.3124	33.1712	0.9923	0.9451
H 05	53.8344	1.5057	4.4435	15.7462	0.9732	0.9093
Hw 03	335.2463	2.6859	63.4549	32.2045	0.9866	0.9489
Hw 04	523.6375	3.6290	156.9179	30.5724	0.9845	0.9154

Hw 05	414.8637	7.2201	74.9028	52.8651	0.9911	0.8742
Hw 06	865.3743	3.0457	275.3415	27.1237	0.9868	0.9352
I 01	381.6416	6.4239	52.9336	84.5937	0.9886	0.9336
I 02	251.6319	5.5728	26.3236	86.1026	0.9906	0.9559
I 03	832.7973	0.3395	20.6710	13.6196	0.9854	0.9523
I 04	118.2232	5.0067	24.1761	53.5882	0.9884	0.9385
I 05	39.6387	5.7866	8.8788	46.0830	0.9912	0.9293
In 01	12.5099	2.4489	7.0419	19.6398	0.9821	0.9486
In 02	14.2350	3.6192	4.8314	26.4565	0.9722	0.9583
In 03	10.0663	2.8763	5.5343	24.6120	0.9805	0.9565
In 04	9.5023	2.5123	3.8019	31.2883	0.9712	0.9574
In 05	232.2849	0.2549	10.9070	21.3090	0.9901	0.8843
Kn 003	90.7911	2.1976	10.4363	106.3604	0.9730	0.9775
Kn 004	22.7563	2.4168	5.7317	48.8011	0.9713	0.9699
Kn 005	133.3226	4.0828	11.0701	162.9435	0.9576	0.9105
Kn 006	60.1808	9.2704	11.7639	184.1050	0.9884	0.9522
Kn 011	51.6393	8.4017	9.1712	169.9285	0.9886	0.9666
Lk 01	13.5811	3.7493	8.3654	16.0503	0.9863	0.9348
Lk 02	107.5765	3.8529	28.6775	25.5444	0.9843	0.9510
Lk 03	58.5481	3.3771	16.6441	21.9140	0.9929	0.9527
Lk 04	20.5803	1.1772	8.9748	10.1449	0.9888	0.9801
Lt 01	423.6948	0.7583	18.3468	32.9066	0.9684	0.9078
Lt 02	751.1495	0.6394	32.6985	31.7868	0.9837	0.9335
Lt 03	88.3092	4.8911	15.2459	101.7612	0.9720	0.9832
Lt 04	95.6604	6.4113	16.2275	101.1050	0.9895	0.9463
Lt 05	33.0904	3.2367	6.6660	52.5286	0.9862	0.9713
Lt 06	12.6603	4.4508	5.3476	33.0414	0.9667	0.9325
M 01	171.7642	4.3245	19.7968	52.0033	0.9879	0.9225
M 02	533.0407	0.5286	48.7393	14.2415	0.9791	0.9693
M 03	146.7155	3.2391	21.3397	36.0318	0.9944	0.9557
M 04	419.3777	0.6036	60.6588	11.5735	0.9666	0.9763
M 05	252.6075	4.1815	46.6180	45.1287	0.9947	0.9306
Mq 01	951.7665	0.5637	89.5533	17.7090	0.9863	0.9588
Mq 02	40.6507	1.5969	20.4135	12.6506	0.9926	0.9655
Mq 03	353.1811	1.7158	22.5446	31.8669	0.9931	0.9856
Mr 001	88.2739	2.6860	13.1552	87.4096	0.9842	0.9815
Mr 018	155.3090	2.1488	24.7275	67.2207	0.9799	0.9863
Mr 026	68.5407	7.3158	13.1804	115.1534	0.9863	0.9633
Mr 027	90.9947	4.9799	20.1624	107.3354	0.9846	0.9456
Mr 288	77.1276	5.0675	15.4370	92.4166	0.9750	0.9683

R 01	58.4179	3.4730	17.2254	36.6148	0.9745	0.9571
R 02	143.5423	1.5010	37.3312	19.1053	0.9780	0.9802
R 03	150.1334	0.8192	20.1034	20.6761	0.9820	0.9778
R 04	53.0530	2.7114	10.7788	39.8437	0.9919	0.9798
R 05	52.8419	1.3771	19.3840	18.2109	0.9783	0.9743
R 06	780.6832	0.2459	16.7087	14.2653	0.9835	0.9350
Rt 01	149.5175	2.2242	20.1494	24.3286	0.9875	0.9645
Rt 02	63.7045	3.7346	19.8587	21.9803	0.9946	0.9316
Rt 03	66.1102	3.1313	18.7931	27.3862	0.9844	0.9465
Rt 04	51.4438	3.1368	14.4738	21.9869	0.9873	0.9358
Rt 05	67.5625	2.8563	25.8076	27.2637	0.9950	0.9469
Ru 01	32.4246	3.8477	5.9434	38.7897	0.9894	0.9604
Ru 02	164.8501	2.2460	23.2986	39.4434	0.9829	0.9915
Ru 03	159.8507	1.4801	59.9352	23.2243	0.9750	0.9620
Ru 04	1238.3964	0.4439	101.5338	20.6088	0.9614	0.9571
Ru 05	729.2764	3.9273	78.0923	76.1505	0.9823	0.9807
Sl 01	74.4823	1.4018	9.1997	24.1027	0.9873	0.9760
Sl 02	78.1667	1.7823	19.2220	32.4544	0.9916	0.9823
Sl 03	125.5762	3.3095	11.1064	60.4119	0.9899	0.9604
Sl 04	466.1931	2.0011	36.1672	36.6358	0.9900	0.9912
Sl 05	205.4954	4.8968	27.2849	75.8361	0.9905	0.9490
Sm 01	184.5525	1.8381	55.9253	15.6411	0.9927	0.9678
Sm 02	111.5533	3.5886	20.4309	29.3903	0.9910	0.9450
Sm 03	39.1556	7.4565	9.0093	24.0300	0.9928	0.8708
Sm 04	85.2935	2.8670	18.1053	21.9164	0.9961	0.9563
Sm 05	27.5367	1.6852	25.0664	11.8320	0.9930	0.9263
T 01	105.9746	5.7859	7.9899	49.1539	0.9887	0.8817
T 02	135.6945	5.2386	8.7591	53.5378	0.9759	0.8685
T 03	136.0871	6.1231	14.8813	41.0608	0.9907	0.8923

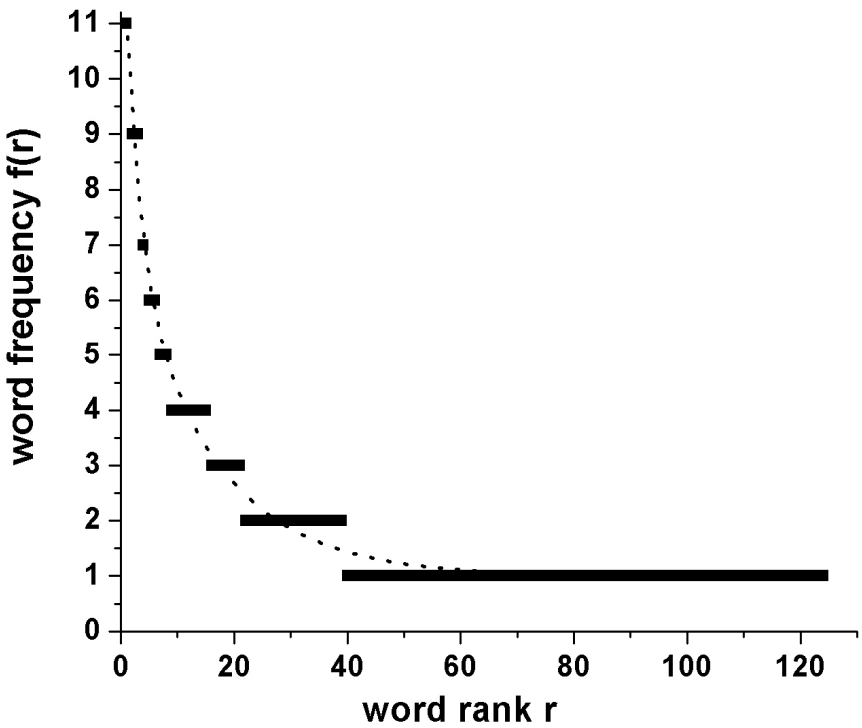


Figure 3.1. Two exponential component fitting of (3.2)
to word rank-frequencies of Goethe's Erlkoenig .
(the fitting data are given in the row (G 17) of Table 1)

In order to compare the results of fitting (3.2) with Zipf´s zeta sequence, the determination coefficient has been computed for both sequences. In spite of all critics, Zipf´s intuition was excellent. The present approach improves slightly his results and this is due not only to the increased number of parameters but to the fact that approach (3.2) takes stratification into account. For a more lucid comparison the determination coefficients of both approaches are presented graphically in Figure 3.2. As can be seen, Zipf´s approach almost always attains $R^2 > 0.9$ but (3.2) is in all cases better and is always greater than 0.95.

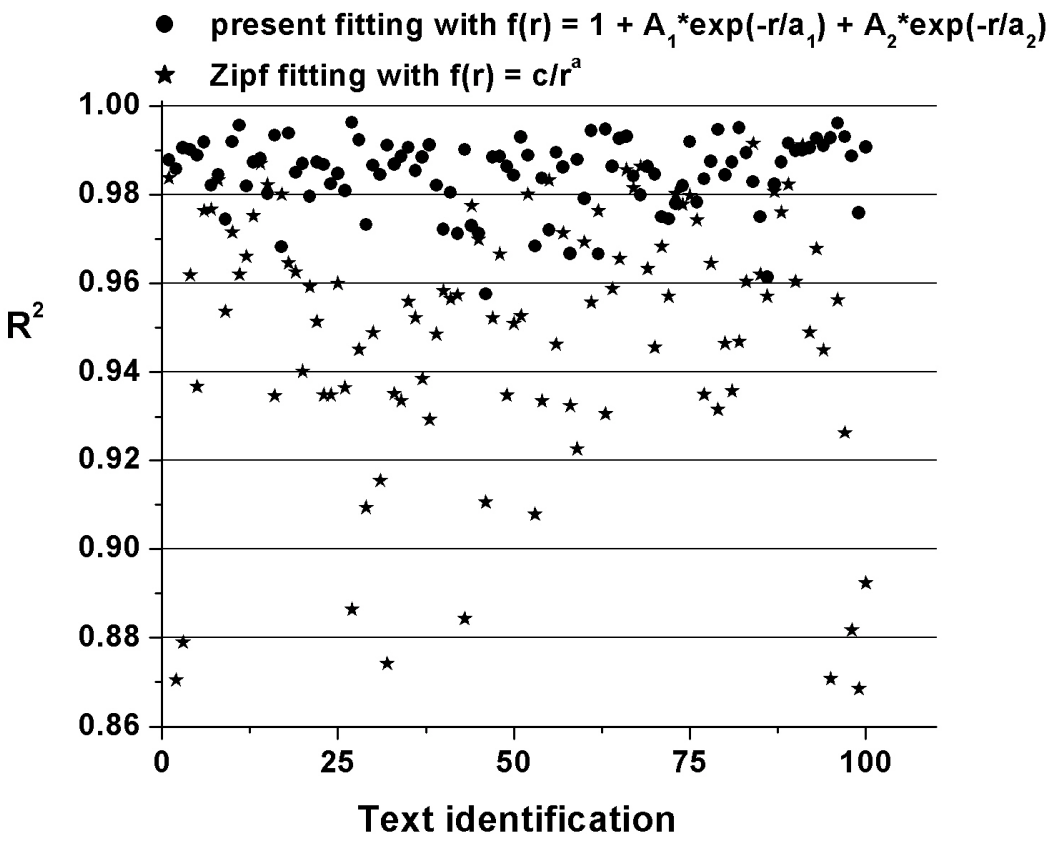


Figure 3.2. Comparing the present fit with the Zipfian (the abscissa does not represent a variable but the number of the text from Table 3.1)

In all cases also three components of (3.2) have been taken into account but not always an improvement has been achieved. In 27 cases out of 100 the fitting with two and three components of (3.2) yielded the same result as shown in Table 3.2.

Table 3.2
Cases in which the determination coefficients for fitting (3.2) with two and three components are identical

ID	R^2		ID	R^2
B 02	0.9858		Kn 005	0.9576
B 03	0.9905		Lk 01	0.9863
E 01	0.9956		M 01	0.9879
G 05	0.9938		M 03	0.9944
G 10	0.9870		M 05	0.9947
G 11	0.9796		Rt 01	0.9875
G 14	0.9867		Rt 02	0.9946

G 17	0.9824		Ru 01	0.9894
H 05	0.9732		Sl 03	0.9899
Hw 03	0.9866		Sm 03	0.9928
In 01	0.9821		Sm 04	0.9961
In 03	0.9805		T 01	0.9887
In 04	0.9712		T 02	0.9759
In 05	0.9901			

However, even in cases when the third component brings a certain improvement, it is usually irrelevant in statistical sense but perhaps relevant philologically. The overall mean R^2 = 0.9848 for fitting with two components and R^2 = 0.9911 for fitting with three components differ merely by 0.0063.

Table 3.3 shows some results and the identity of parameters up to 4 components. In some of the texts up to 4 strata can be traced down. If a stratum is not present, the exponential parameter equals one of the preceding ones. This is a unique method for a deeper insight in the text forming and a challenge for philologists. From the statistical point of view, maximally two components of (3.2) are necessary in order to capture any word-form rank-frequency sequence.

Rank frequency sequences exist in all domains of language, even at the lowest level, with sounds or phonemes. One can consider e.g. phonemes as a closed class of equivalent entities or partition the inventory in vowels, consonants, semivowels, diphthongs, glides etc. As is well known, ranked frequencies of phonemes do not always display smooth monotonously decreasing sequences. The above technique could give hints at the stratification of contrasts.

Table 3.3
Fitting (3.2) with 2, 3 and 4 components to German and English texts
(Gray highlighted: double or triple identical data on the same row)

	2Exp Fit	**3Exp Fit**	**4Exp Fit**									**number of strata**
ID	**R^2**	**R^2**	**R^2**	A_1	a_1	A_2	a_2	A_3	a_3	A_4	a_4	
E 01	0.9956	0.9956	0.9956	68.5431	3.7495	45.3579	3.7496	28.8594	3.7489	16.3721	53.19016	2
G 10	0.9870	0.9870	0.9870	9.7043	4.1381	4.6573	26.4227	4.0456	4.1354	3.5621	4.1356	2
G 14	0.9867	0.9867	0.9867	5.3719	1.5978	5.0782	7.6284	1.4684	7.6203	0.4996	7.6247	2
G 17	0.9824	0.9824	0.9824	4.7168	14.9218	3.8320	2.4555	2.3345	2.4550	1.6721	14.8977	2
E 04	0.9881	0.9948	0.9948	438.2241	1.8032	75.8346	1.8064	66.5780	13.4783	13.4465	133.6564	3
E 07	0.9933	0.9957	0.9957	144.6868	4.4323	103.7906	4.3847	42.0299	32.4846	5.28151	216.0463	3
G 05	0.9938	0.9938	0.9939	84.6561	0.2278	15.8915	3.5524	15.3136	3.5524	4.5458	30.4477	3
G 09	0.9850	0.9907	0.9907	1566.4351	0.1612	1559.6730	0.1611	20.0090	4.8094	6.66207	28.7125	3
G 11	0.9796	0.9796	0.9800	12.0612	1.7579	6.7582	4.6103	5.16337	25.8110	0.38219	4.6187	3
G 12	0.9873	0.9883	0.9883	249.3094	0.2329	5.4186	10.1754	4.1267	2.6324	2.0578	10.0010	3
E 02	0.9819	0.9964	0.9976	7881.5540	0.2034	82.9117	4.6010	35.6566	22.8173	8.4227	92.4218	4
E 03	0.9873	0.9926	0.9935	176.9265	1.2423	143.3683	4.2644	33.1930	21.4565	6.3083	142.1664	4
E 05	0.9802	0.9918	0.9938	386.9936	0.9278	148.0335	5.5419	37.7644	30.4036	6.0687	204.2925	4
E 13	0.9682	0.9938	0.9963	14853.7474	0.2560	465.9355	5.4046	78.3550	37.8173	16.6900	246.4941	4

4. The h-point

The *h*-point can be defined as that point at which the straight line between two (usually) neighbouring ranked frequencies intersects the $y = x$ line. Solving two simultaneous equations we obtain the definition

$$(4.1)\quad h = \begin{cases} r, & \text{if there is an } r = f(r) \\ \dfrac{f(i)r_j - f(j)r_i}{r_j - r_i + f(i) - f(j)}, & \text{if there is no } r = f(r) \end{cases}.$$

In other words, the *h*-point is that point at which $r = f(r)$. If there is no such point, one takes, if possible, two neighbouring $f(i)$ and $f(j)$ such that $f(i) > r_i$ and $f(j) < r_j$. Mostly $r_i + 1 = r_j$. As an example consider the last column in Table 2.2 in Chapter 2 where we have

r	1,	2,	3,	4,	5
$f(r)$	4,	2,	1,	1,	1

Here, evidently $h = 2$ because the frequency at rank 2 is 2. However, the overall sequence of frequencies in Chapter 2 is

r	1,	2,	3,	4,	5,	6,	7 ,	8,	9,	10,	11,	12,	13,	14,	15,	16,	17,	18,	19,	20,	21,	22,	23,	24,	25,	26,	27,	28,	29
$f(r)$	30,	20,	16,	12,	12,	8,	8,	7,	5,	5,	2,	2,	2,	2,	1,	1,	1,	1,	1,	1,	1,	1,	1,	1,	1,	1,	1,	1,	1.

Here, there is no $r = f(r)$, but $r = 7$, $f(7) = 8$ and $r = 8$, $f(8) = 7$ fulfil the above condition. Inserting these values in the second part of (4.1) we obtain

$$h = \frac{f(7)8 - f(8)7}{8 - 7 + f(7) - f(8)} = \frac{8(8) - 7(7)}{8 - 7 + 8 - 7} = 7.5,$$

here the average of two ranks, but it need not be always the case. Other computing possibilities have been presented in the extensive literature. The *h*-point has been created in scientometrics by Hirsch (2005) and discussed there and in documentation mathematically (cf. e.g. Bornmann, Daniel 2005; Egghe 2007a,b, 2008; Egghe, Rao 2008; Egghe, Rousseau 2006; Rousseau 2007; Rousseau, Liu 2008); in linguistics it appeared for the first time in Popescu (2007). This remarkable point is an attractive fixed-point having different uses in textology (cf. http://en.wikipedia.org/wiki/ Fixed_point_%28mathematics%29).

In other applications, e.g. parts of speech, where the inventory is too small, neither of the two conditions may be fulfilled. For example in the sequence

r	1,	2,	3,	4,	5,	6
$f(r)$	100,	80,	60,	50,	40,	30

where all $f(r) > r$. The problem can be solved in different ways but we prefer the transformation

(4.2) $f^*(r) = f(r) - f(V) + 1$, for all r,

where V is the greatest rank, warranting that $f^*(V) = 1 < V$. For the above sequence we obtain

100 - 30 + 1 = 71 etc.

hence

r	1,	2,	3,	4,	5,	6
$f^*(r)$	71,	51,	31,	21,	11,	1

where $r = 5$ and 6 fulfil the above condition and one obtains $h = 5.55$.

The h-point seems to be an important indicator in rank-frequency phenomena. As is well known, every text consists of autosemantics which bring up the theme and the concomitant information, and of synsemantics which care for correct relations between autosemantics and sentences, furnish references and modify the autosemantics. The number of synsemantics is always greater than that of autosemantics, and usually they occupy the first ranks. The h-point forms a fuzzy threshold between these two kinds of words. Of course, some synsemantics seldom occur – depending on style – and occupy some higher ranks. On the other hand, some autosemantics may occur more frequently than $f(h)$ and their occurrence in the pre-h domain signalizes their association to the theme of the text. In fiction one often finds proper names in the pre-h domain but in scientific and technical texts these words are always thematic words. The more autosemantics are in this domain and the more frequent they are, the greater the thematic concentration of the text. Popescu and Altmann (2007a) proposed the *indicator of thematic concentration* in form

(4.3) $$TC = 2\sum_{r'=1}^{h} \frac{(h - r')\, f(r')}{h(h-1)\, f(1)},$$

where r' are the ranks of autosemantics occupying ranks smaller than h. Here the difference between the pertinent ranks is weighted by the given frequency, and the sum of the differences is divided by the possible maximum. Since TC is a very small number, one usually multiplies it by a constant, e.g. 1000 and obtains a thematic concentration unit $tcu = 1000(TC)$. A survey of tcu-values can be

found in the book Popescu et al. (2009) and a recent extension of the *TC* concept in Tuzzi, Popescu, Altmann (2009).

Some further useful indicators and functions associated to the *h*-point are the crowding of autosemantics, autosemantic pace filling, autosemantic compactness and writer´s view (cf. Popescu 2007; Popescu, Altmann 2006a,b, 2007; Popescu, Best, Altmann 2007; Popescu et al. 2008; Mačutek, Popescu, Altmann 2007) all of which can be used for stylistic analyses and can provide us with a deeper insight in the structure of texts.

Here we shall consider the angles of the rank-frequency distribution and present bi- and triangular text and language classifications.

Three relevant angular fields imply the *h*-point, as shown in Figure 1, namely the "writer's view" angle α between the distribution end (P_1) and top (P_2) as seen from the *h*-point (H), baptized in this way because one can imagine the writer "sitting" at this point and controlling the equilibrium between autosemantics and synsemantics (cf. Popescu, Altmann 2007); the view angle β of the autosemantic arch span (P_1H) as seen from the top (P_2), and the view angle γ of the synsemantic arch span (P_2H) as seen from the end (P_1). One can imagine these angles as word-frequency self-regulation means. They are unconscious but they care for shaping the frequencies. Texts can attain some extreme points, but the variation is very restricted.

Let us further derive the expressions of the natural cosines of these angles ("natural" means in radians) starting from the general expression of the scalar (dot) product of two vectors **a** (a_x, a_y) and **b** (b_x, b_y), namely

$$\cos\alpha = \frac{\mathbf{a}\bullet\mathbf{b}}{ab} = \frac{a_x b_x + a_y b_y}{\left[\left(a_x^{\,2} + a_y^{\,2}\right)^{1/2}\right]\left[\left(b_x^{\,2} + b_y^{\,2}\right)^{1/2}\right]} \tag{4.4}$$

and from the particular coordinates of the corners of the distribution characteristic triangle P_1P_2H given by

$P_1\,(V, 1)$
$P_2\,(1, f(1))$
$H\,(h, h)$

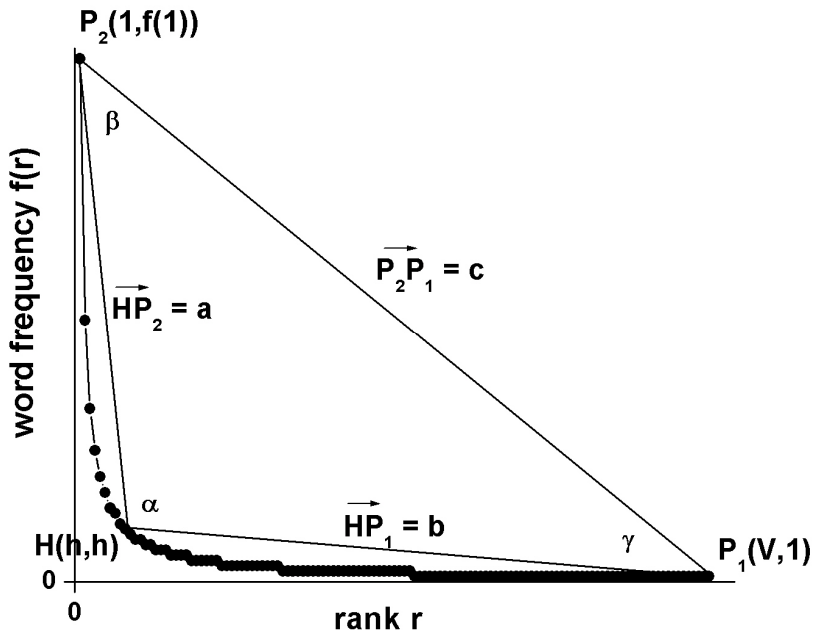


Figure 4.1. Characteristic α, β and γ view angles associated to the *h*-point

respectively from the particular vectors

a ($a_x = -(h-1)$; $a_y = f(1) - h$) directed from H to P_2
b ($b_x = V - h$; $b_y = -(h-1)$) directed from H to P_1
c ($c_x = (V-1)$; $c_y = -(f(1)-1)$) directed from P_2 to P_1

Thus, for "writer's view" cos α we get[1]

(4.5)

$$\cos\alpha = \frac{\mathbf{a}\bullet\mathbf{b}}{ab} = \frac{-\left[(h-1)\left(f(1)-h\right)+(h-1)(V-h)\right]}{\left[(h-1)^2+\left(f(1)-h\right)^2\right]^{1/2}\left[(h-1)^2+\left(V-h\right)^2\right]^{1/2}}$$

and, similarly, for the "autosemantics view" cos β

[1] Actually, the original writer's view formula (Popescu, Altmann 2007) differs from Eq. (4.5) by merely replacing (h – 1) through h so that both expressions give practically the same results for $h >> 1$.

$$(4.6)\quad \cos b = \frac{-\mathbf{a}\bullet\mathbf{c}}{ac} = \frac{(h-1)(V-1) + (f(1)-1)(f(1)-h)}{\left[(h-1)^2+(f(1)-h)^2\right]^{1/2}\left[(V-1)^2+(f(1)-1)^2\right]^{1/2}}$$

and for the "synsemantics view" cos γ

$$(4.7)\quad \cos\gamma = \frac{\mathbf{b}\bullet\mathbf{c}}{bc} = \frac{(V-1)(V-h) + (h-1)(f(1)-1)}{\left[(V-1)^2+(f(1)-1)^2\right]^{1/2}\left[(h-1)^2+(V-h)^2\right]^{1/2}}$$

Finally, it should be noted that the angle sum is always subjected to the geometrical condition $\alpha + \beta + \gamma = \pi$, hence only two angles are necessary to fix the triangle shape.

The view angles α, β, γ (in radians) of 176 texts in 20 languages as computed with the above formulas are given in Table 4.1 and Table 4.2 (for language averages).[2] The resulting relevant graphs are presented in Figure 4.2 for the angle dependence on the text size N, in Figures 4.3, 4.6, and 4.9 respectively for the bi-angular $<\alpha,\beta>$, $<\alpha,\gamma>$, and $<\gamma,\beta>$ text classification, in Figures 4.4, 4.7, and 4.10 for the corresponding suggested models, and in Figures 4.5, 4.8, and 4.11 respectively for language averages.

Table 4.1
Three angle views α, β, γ (in radians) of 176 texts in 20 languages
(N = text length, V = text vocabulary, $f(1)$ = the greatest frequency in text, h-point rounded to integer)

Text ID	N	V	$f(1)$	h	α	β	γ
B 01	761	400	40	10	1.8853	1.1819	0.0744
B 02	352	201	13	8	2.5576	0.5603	0.0237
B 03	515	285	15	9	2.5271	0.5942	0.0203
B 04	483	286	21	8	2.0899	1.0068	0.0449
B 05	406	238	19	7	2.0604	1.0313	0.0498
B 06	687	388	28	9	1.9904	1.1026	0.0485
B 07	557	324	19	8	2.1597	0.9484	0.0335
B 08	268	179	10	6	2.4957	0.6242	0.0216
B 09	550	313	20	9	2.2259	0.8812	0.0345
B 10	556	317	26	7	1.8960	1.1860	0.0596
Cz 01	1044	638	58	9	1.7454	1.3197	0.0765

[2] The data were taken from Popescu et al. (2009) . The languages used are: B = Bulgarian, Cz = Czech, E = English, G = German, H = Hungarian, Hw = Hawaiian, I = Italian, In = Indonesian, Kn = Kannada, Lk = Lakota, Lt = Latin, M = Maori, Mq = Marquesan, Mr = Marathi, R = Romanian, Rt = Rarotongan, Ru = Russian, Sl = Slovenian, Sm = Samoan, T = Tagalog.

Cz 02	984	543	56	11	1.8083	1.2510	0.0823
Cz 03	2858	1274	182	19	1.6951	1.3196	0.1269
Cz 04	522	323	27	7	1.8812	1.1988	0.0616
Cz 05	999	556	84	9	1.6917	1.3161	0.1338
Cz 06	1612	840	106	13	1.7136	1.3180	0.1100
Cz 07	2014	862	134	15	1.7044	1.3004	0.1367
Cz 08	677	389	31	8	1.8846	1.1982	0.0588
Cz 09	460	259	30	6	1.7960	1.2535	0.0922
Cz 10	1156	638	50	11	1.8377	1.2430	0.0608
E 01	2330	939	126	16	1.7226	1.3028	0.1162
E 02	2971	1017	168	22	1.7348	1.2650	0.1418
E 03	3247	1001	229	19	1.6746	1.2611	0.2058
E 04	4622	1232	366	23	1.6530	1.2185	0.2701
E 05	4760	1495	297	26	1.6798	1.2832	0.1786
E 06	4862	1176	460	24	1.6435	1.1457	0.3524
E 07	5004	1597	237	25	1.6988	1.3113	0.1315
E 08	5083	985	466	26	1.6536	1.0726	0.4154
E 09	5701	1574	342	29	1.6781	1.2681	0.1954
E 10	6246	1333	546	28	1.6436	1.1303	0.3677
E 11	8193	1669	622	32	1.6422	1.1619	0.3375
E 12	9088	1825	617	39	1.6577	1.1795	0.3044
E 13	11265	1659	780	41	1.6496	1.0775	0.4145
G 01	1095	530	83	12	1.7457	1.2633	0.1326
G 02	845	361	48	9	1.7958	1.2387	0.1071
G 03	500	281	33	8	1.8694	1.1840	0.0882
G 04	545	269	32	8	1.8814	1.1718	0.0883
G 05	559	332	30	8	1.9005	1.1754	0.0658
G 06	545	326	30	8	1.9009	1.1737	0.0670
G 07	263	169	17	5	1.9169	1.1541	0.0706
G 08	965	509	39	11	1.9339	1.1531	0.0546
G 09	653	379	30	9	1.9564	1.1302	0.0550
G 10	480	301	18	7	2.0905	1.0148	0.0362
G 11	468	297	18	7	2.0908	1.0141	0.0367
G 12	251	169	14	6	2.1601	0.9350	0.0466
G 13	460	253	19	8	2.1661	0.9328	0.0427
G 14	184	129	10	5	2.2778	0.8259	0.0380
G 15	593	378	16	8	2.3085	0.8122	0.0209
G 16	518	292	16	8	2.3143	0.8005	0.0269
G 17	225	124	11	6	2.3985	0.7043	0.0388
H 01	2044	1079	225	12	1.6327	1.3143	0.1946
H 02	1288	789	130	8	1.6371	1.3512	0.1533
H 03	403	291	48	4	1.6493	1.3420	0.1502

H 04	936	609	76	7	1.6675	1.3613	0.1128
H 05	413	290	32	6	1.7784	1.2740	0.0893
Hw 01	282	104	19	7	2.0962	0.9341	0.1112
Hw 02	1829	257	121	21	1.8527	0.9351	0.3538
Hw 03	3507	521	277	26	1.7205	0.9836	0.4375
Hw 04	7892	744	535	38	1.6975	0.8733	0.5708
Hw 05	7620	680	416	38	1.7259	0.9246	0.4910
Hw 06	12356	1039	901	44	1.6641	0.8064	0.6711
I 01	11760	3667	388	37	1.6829	1.3634	0.0953
I 02	6064	2203	257	25	1.6849	1.3520	0.1047
I 03	854	483	64	10	1.7550	1.2757	0.1109
I 04	3258	1237	118	21	1.7906	1.2731	0.0779
I 05	1129	512	42	12	1.9442	1.1393	0.0581
In 01	376	221	16	6	2.0577	1.0391	0.0448
In 02	373	209	18	7	2.0998	0.9899	0.0519
In 03	347	194	14	6	2.1560	0.9449	0.0407
In 04	343	213	11	5	2.1780	0.9357	0.0279
In 05	414	188	16	8	2.3285	0.7719	0.0412
Kn 003	3188	1833	74	13	1.7716	1.3367	0.0332
Kn 004	1050	720	23	7	1.9380	1.1814	0.0222
Kn 005	4869	2477	101	16	1.7516	1.3558	0.0343
Kn 006	5231	2433	74	20	1.9170	1.2025	0.0221
Kn 011	4541	2516	63	17	1.9119	1.2114	0.0182
Kn 012	4141	1842	58	19	2.0131	1.1074	0.0211
Kn 013	1302	807	35	10	1.9276	1.1831	0.0309
Kn 016	4735	2356	93	18	1.8010	1.3089	0.0318
Kn 017	4316	2122	122	18	1.7409	1.3518	0.0489
Lk 01	345	174	20	8	2.1410	0.9333	0.0672
Lk 02	1633	479	124	17	1.7538	1.1705	0.2172
Lk 03	809	272	62	12	1.8296	1.1328	0.1791
Lk 04	219	116	18	6	2.0110	1.0292	0.1013
Lt 01	3311	2211	133	12	1.6665	1.4205	0.0547
Lt 02	4010	2334	190	18	1.6767	1.3914	0.0735
Lt 03	4931	2703	103	19	1.7886	1.3220	0.0310
Lt 04	4285	1910	99	20	1.8169	1.2835	0.0412
Lt 05	1354	909	33	8	1.8516	1.2626	0.0275
Lt 06	829	609	19	7	2.0444	1.0776	0.0196
M 01	2062	398	152	18	1.7417	1.0811	0.3187
M 02	1175	277	127	15	1.7485	1.0182	0.3749
M 03	1434	277	128	17	1.7754	0.9964	0.3698
M 04	1289	326	137	15	1.7300	1.0602	0.3513
M 05	3620	514	234	26	1.7416	1.0248	0.3751

Mq 01	2330	289	247	22	1.7424	0.7708	0.6284
Mq 02	457	150	42	10	1.9092	1.0281	0.2043
Mq 03	1509	301	218	14	1.6797	0.8809	0.5809
Mr 001	2998	1555	75	14	1.7892	1.3132	0.0391
Mr 002	2922	1186	73	18	1.8851	1.2103	0.0461
Mr 003	4140	1731	68	20	1.9588	1.1552	0.0276
Mr 004	6304	2451	314	24	1.6594	1.3646	0.1176
Mr 005	4957	2029	172	19	1.6969	1.3696	0.0752
Mr 006	3735	1503	120	19	1.7593	1.3154	0.0669
Mr 007	3162	1262	80	16	1.8131	1.2780	0.0505
Mr 008	5477	1807	190	27	1.7436	1.3083	0.0897
Mr 009	6206	2387	93	26	1.9385	1.1751	0.0280
Mr 010	5394	1650	217	27	1.7228	1.3046	0.1142
Mr 015	4693	1947	136	21	1.7534	1.3293	0.0589
Mr 016	3642	1831	63	18	1.9414	1.1757	0.0245
Mr 017	4170	1853	67	19	1.9394	1.1764	0.0258
Mr 018	4062	1788	126	20	1.7589	1.3236	0.0591
Mr 020	3943	1825	62	19	1.9772	1.1409	0.0235
Mr 021	3846	1793	58	20	2.0452	1.0754	0.0211
Mr 022	4099	1703	142	21	1.7465	1.3243	0.0708
Mr 023	4142	1872	72	20	1.9314	1.1826	0.0277
Mr 024	4255	1731	80	20	1.8886	1.2185	0.0345
Mr 026	4146	2038	84	19	1.8499	1.2599	0.0318
Mr 027	4128	1400	92	21	1.8599	1.2313	0.0505
Mr 028	5191	2386	86	23	1.9161	1.1992	0.0263
Mr 029	3424	1412	28	17	2.5508	0.5832	0.0077
Mr 030	5504	2911	86	20	1.8577	1.2613	0.0226
Mr 031	5105	2617	91	21	1.8568	1.2581	0.0267
Mr 032	5195	2382	98	23	1.8655	1.2448	0.0314
Mr 033	4339	2217	71	19	1.9122	1.2060	0.0234
Mr 034	3489	1865	40	17	2.1873	0.9421	0.0123
Mr 035	1862	1115	29	11	2.0870	1.0386	0.0161
Mr 036	4205	2070	96	19	1.8092	1.2953	0.0371
Mr 038	4078	1607	66	20	1.9745	1.1386	0.0285
Mr 040	5218	2877	81	21	1.8995	1.2212	0.0208
Mr 043	3356	1962	44	16	2.0703	1.0571	0.0142
Mr 046	4186	1458	68	20	1.9609	1.1479	0.0327
Mr 052	3549	1628	89	17	1.7994	1.2981	0.0441
Mr 149	2946	1547	47	12	1.8825	1.2365	0.0226
Mr 150	3372	1523	64	16	1.8836	1.2265	0.0314
Mr 151	4843	1702	192	23	1.7133	1.3295	0.0987
Mr 154	3601	1719	68	17	1.8842	1.2278	0.0296

Mr 288	4060	2079	84	17	1.8130	1.2965	0.0322
Mr 289	4831	2312	112	19	1.7698	1.3316	0.0401
Mr 290	4025	2319	42	17	2.1471	0.9838	0.0107
Mr 291	3954	1957	86	18	1.8245	1.2824	0.0347
Mr 292	4765	2197	88	19	1.8342	1.2760	0.0313
Mr 293	3337	2006	41	13	1.9817	1.1460	0.0139
Mr 294	3825	1931	85	17	1.8102	1.2962	0.0351
Mr 295	4895	2322	97	20	1.8210	1.2875	0.0331
Mr 296	3836	1970	92	18	1.8053	1.2988	0.0375
Mr 297	4605	2278	88	18	1.8166	1.2944	0.0307
R 01	1738	843	62	14	1.8510	1.2340	0.0566
R 02	2279	1179	110	16	1.7419	1.3203	0.0794
R 03	1264	719	65	12	1.7910	1.2773	0.0733
R 04	1284	729	49	10	1.8101	1.2782	0.0533
R 05	1032	567	46	11	1.8671	1.2132	0.0614
R 06	695	432	30	10	2.0150	1.0808	0.0459
Rt 01	968	223	111	14	1.7661	0.9775	0.3979
Rt 02	845	214	69	13	1.8415	1.0507	0.2494
Rt 03	892	207	66	13	1.8552	1.0425	0.2439
Rt 04	625	181	49	11	1.8869	1.0529	0.2018
Rt 05	1059	197	74	15	1.8805	0.9813	0.2798
Ru 01	753	422	31	8	1.8831	1.2042	0.0542
Ru 02	2595	1240	138	16	1.7054	1.3383	0.0979
Ru 03	3853	1792	144	21	1.7433	1.3299	0.0684
Ru 04	6025	2536	228	25	1.6980	1.3638	0.0798
Ru 05	17205	6073	701	41	1.6380	1.3955	0.1081
Sl 01	756	457	47	9	1.7961	1.2628	0.0827
Sl 02	1371	603	66	13	1.8138	1.2406	0.0872
Sl 03	1966	907	102	13	1.7182	1.3258	0.0976
Sl 04	3491	1102	328	21	1.6544	1.2170	0.2702
Sl 05	5588	2223	193	25	1.7236	1.3427	0.0753
Sm 01	1487	267	159	17	1.7469	0.9226	0.4721
Sm 02	1171	222	103	15	1.7961	0.9806	0.3649
Sm 03	617	140	45	13	2.0238	0.9055	0.2124
Sm 04	736	153	78	12	1.8138	0.9368	0.3910
Sm 05	447	124	39	11	2.0021	0.9281	0.2114
T 01	1551	611	89	14	1.7642	1.2559	0.1215
T 02	1827	720	107	15	1.7417	1.2734	0.1265
T 03	2054	645	128	19	1.7632	1.2124	0.1660

Table 4.2
Mean view angles of 20 languages

Language	mean α	mean β	mean γ
B Bulgarian	2.1888	0.9117	0.0411
Cz Czech	1.7758	1.2718	0.0940
E English	1.6717	1.2060	0.2639
G German	2.0416	1.0402	0.0597
H Hungarian	1.6730	1.3286	0.1400
Hw Hawaiian	1.7928	0.9095	0.4392
I Italian	1.7715	1.2807	0.0894
In Indonesian	2.1640	0.9363	0.0413
Kn Kannada	1.8636	1.2488	0.0292
Lk Lakota	1.9339	1.0665	0.1412
Lt Latin	1.8074	1.2929	0.0413
M Maori	1.7475	1.0362	0.3580
Mq Marquesan	1.7771	0.8933	0.4712
Mr Marathi	1.8856	1.2171	0.0389
R Romanian	1.8460	1.2339	0.0616
Rt Rarotongan	1.8461	1.0210	0.2746
Ru Russian	1.7336	1.3264	0.0817
Sl Slovenian	1.7412	1.2778	0.1226
Sm Samoan	1.8765	0.9347	0.3303
T Tagalog	1.7564	1.2472	0.1380

Notice the “golden” lower limit 1.618… of the “writer’s view” α angle treated separately in Popescu, Altmann (2008b) and appearing always when the angle α is concerned (cf. for example in Fig. 4.3 and 4.6). Taking only one of the angles in Table 4.2, we do not obtain any reasonable classification of languages. But taking all combinations of two angles separately we obtain very clear attractors which can be characterized for the first in an elementary way. As can be seen in Figure 4.3, the <*α*, *β*> relation is clearly demarcated: the minimum of α is the golden section 1.618…, its maximum is ca 2.6. Evidently some of the texts attain extreme values which can be eliminated as outliers or smoothed in form of averages of the given language.

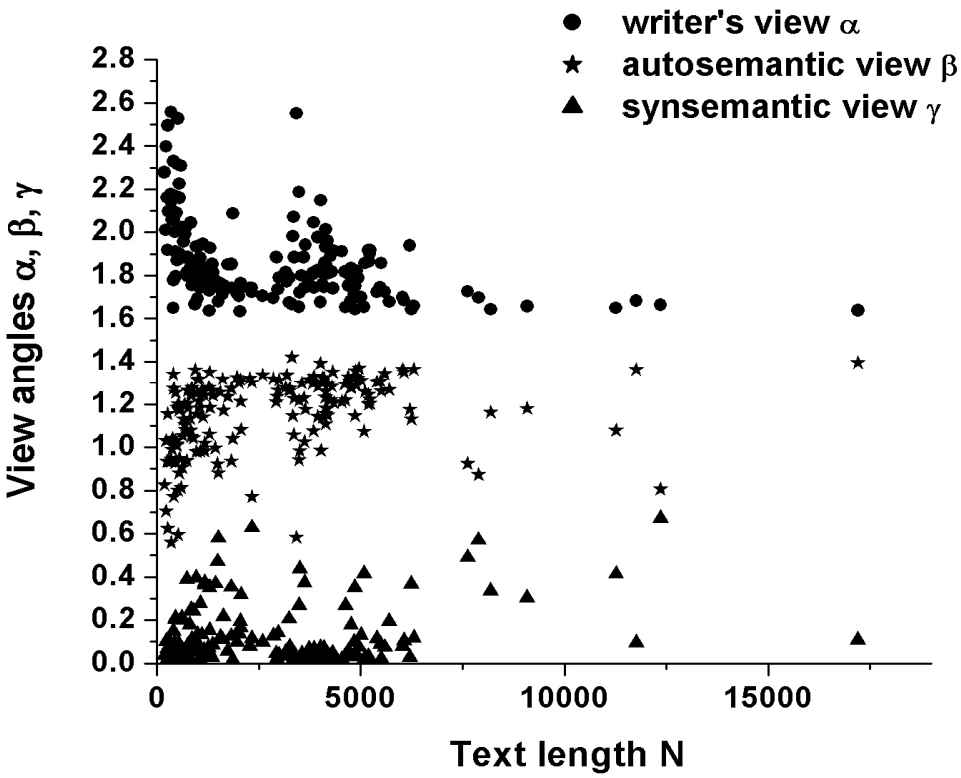


Figure 4.2. View angles α, β, γ in terms of the text length.

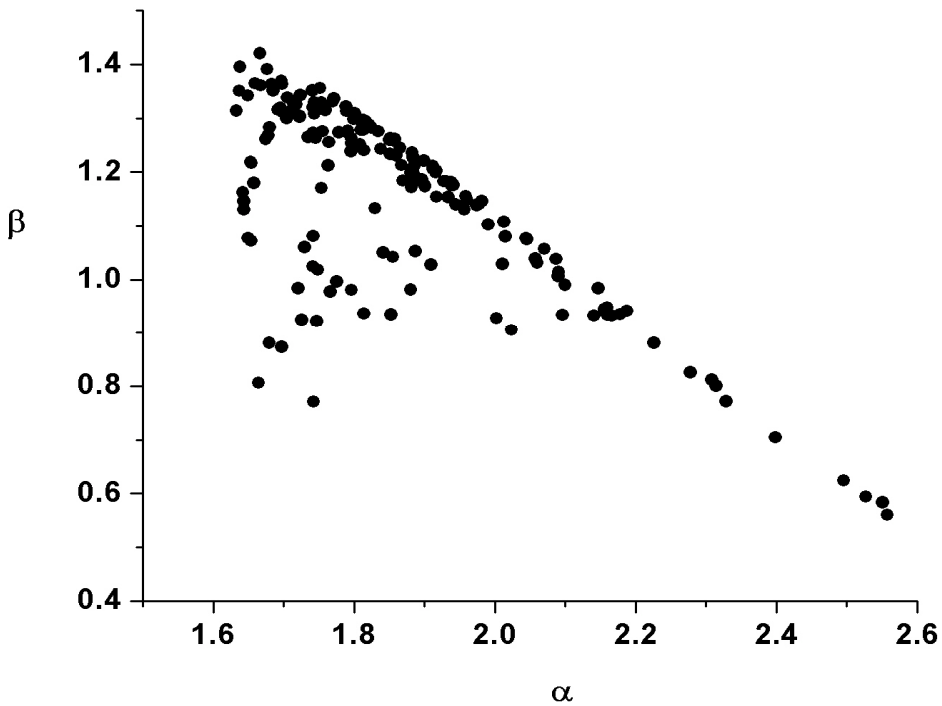


Figure 4.3. Bi-angular <α, β> text classification.

In Figure 4.3, the area of empirically observed texts can be delimited very exactly by the formula

(4.8) $\beta = (\alpha - 1.618)^m(2.56 - \alpha) + 0.56$

which using different *m* shows the individual trends. It would be interesting to study whether languages or different text types prefer narrow domains of this type with *m* in a small interval, or are freely distributed in the whole domain. Further, it will be possible to study whether texts having the same *m* do have some commonalities. The result is shown in Figure 4.4. In formula (4.8) the golden section 1.618 is the lower boundary of the writer's view α, the constant 2.56 is simply the maximum of α-values, and 0.56 the minimum of β-values.

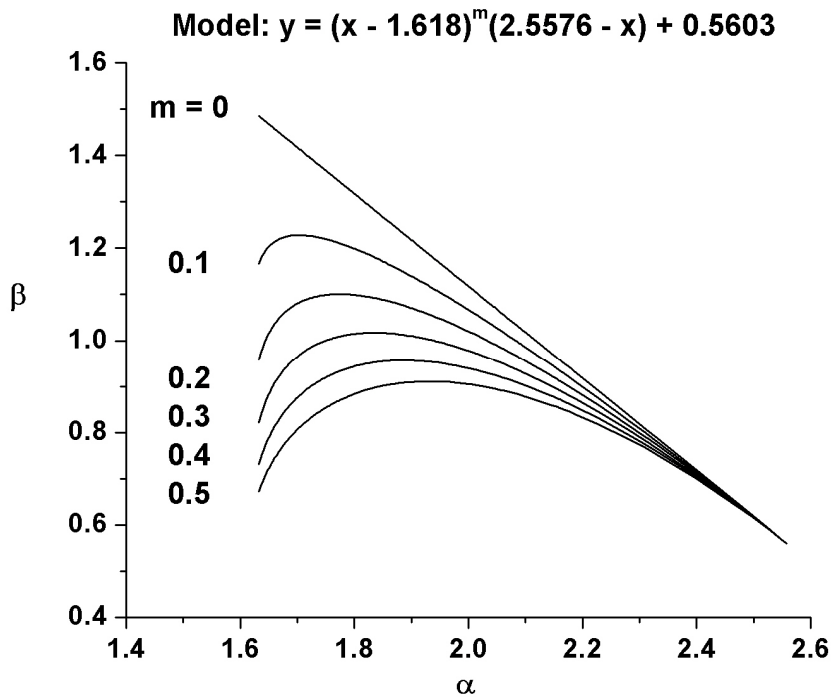


Figure 4.4. The area of <α,β> classification

If we take averages for texts in individual languages, we obtain the situation as presented in Figure 4.5 yielding almost the same view but the extreme lower part disappears.

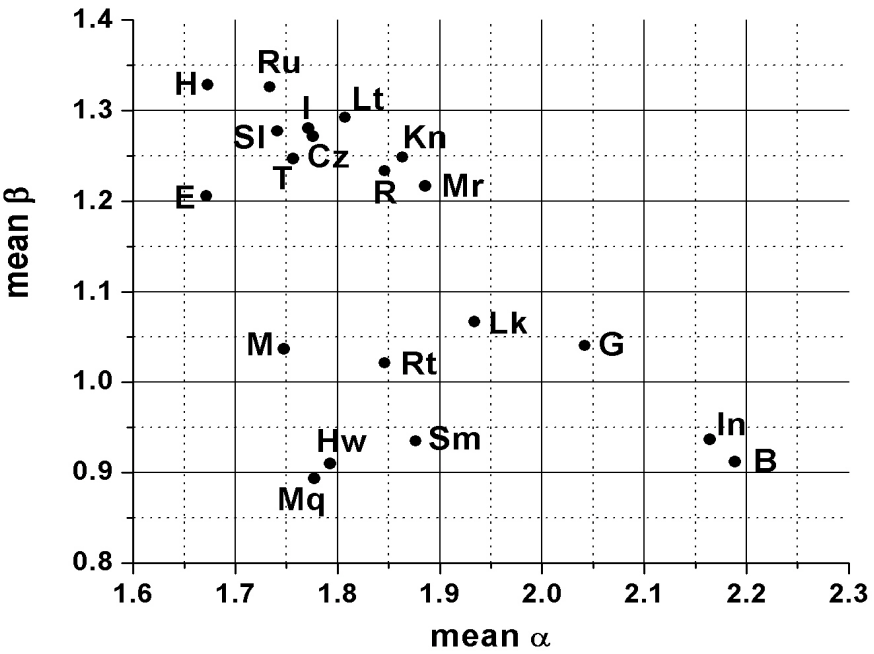


Figure 4.5. Bi-angular <mean α, mean β> language classification

Here the names of languages could be inscribed because we had only 20 points. As one can see, we obtain approximately a triangle. Strongly analytic languages are concentrated rather in the left-bottom corner, more synthetic ones are situated rather in the upper part of the triangle. The result is not final presumably because the existence of factors like style, genre, personality, etc. was not taken in account here. The most surprising is the distance between Tagalog and Indonesian.

The <α,γ> classification yields another picture which apparently might be enclosed by two hyperbolic curves, as can be seen in Figure 4.6. This time these empirical data can be modelled by the equation

(4.9) $\gamma = (\alpha - 1.618)^m(2.56 - \alpha)^2 + 0.01$

as illustrated in Figure 4.7. Here again the golden section 1.618 is the lower boundary of the writer's view α, the constant 2.56 is simply the maximum of α-values, and 0.01 about the minimum of γ -values

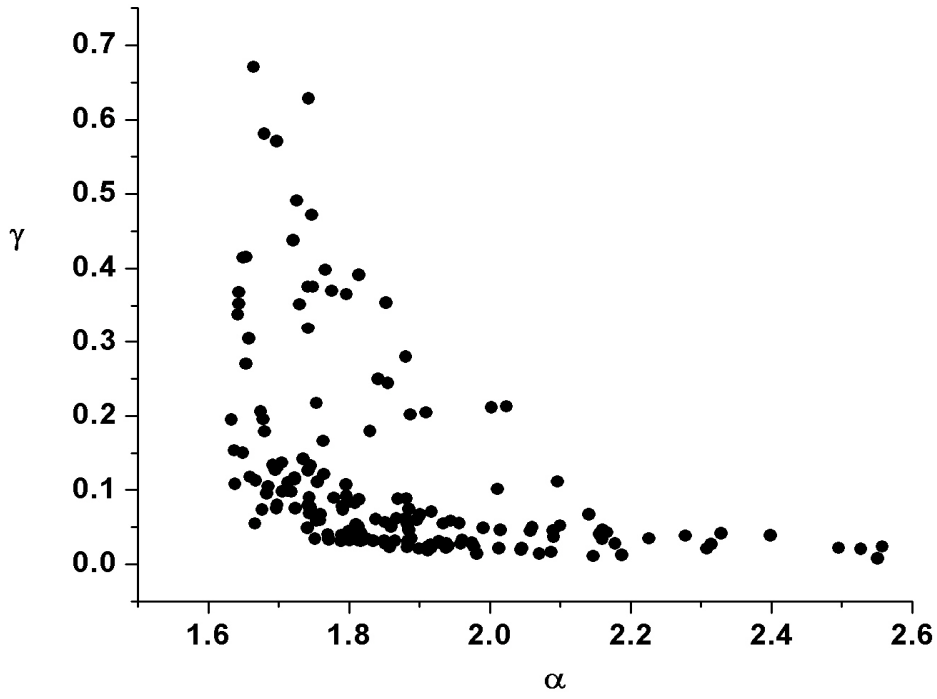


Figure 4.6. Bi-angular <α, γ> text classification.

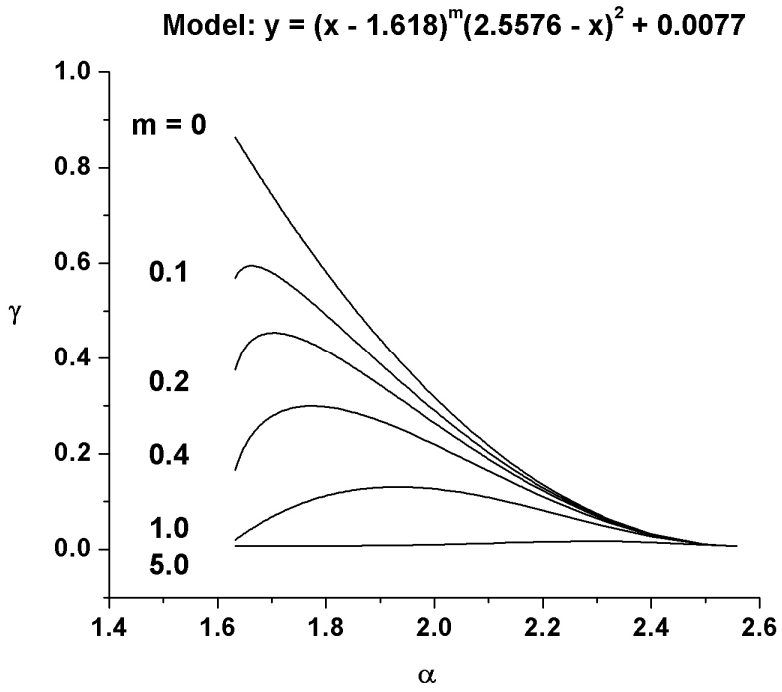


Figure 4.7. The area of <*α, γ*> classification

The use of means yields a transformed picture (cf. Figure 4.8).

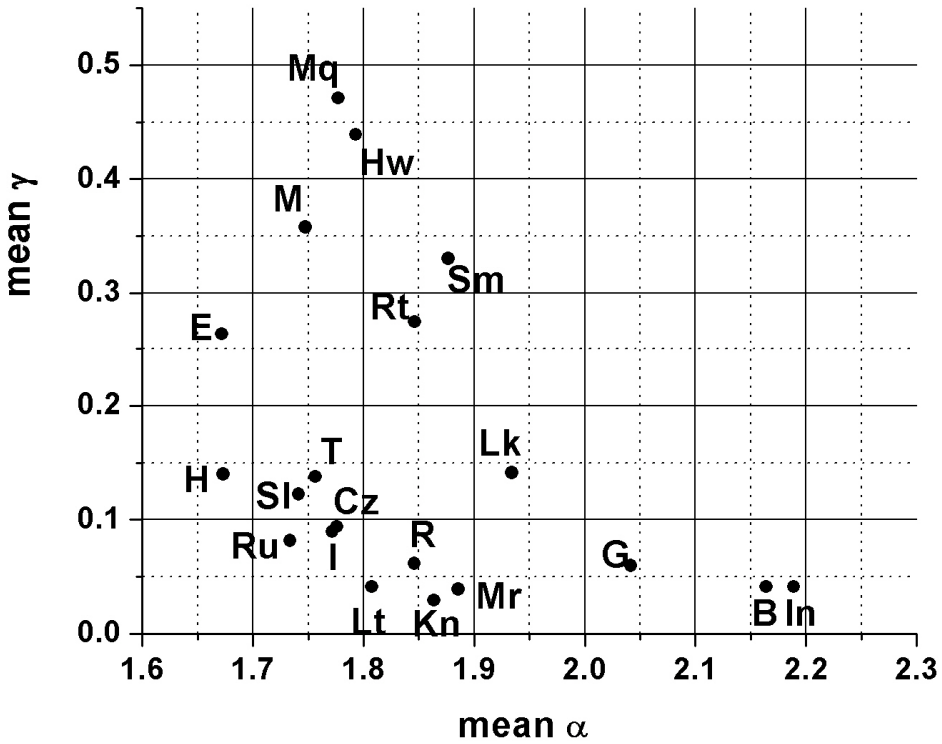


Figure 4.8. Bi-angular <mean α, mean γ> language classification

The same holds for the <γ,β> classification as shown in the following Figures 4.9, 4.10, and 4.11. In this later case the modelling formula is suggested in the form

(4.10) $\beta = (\gamma - 0.01)^m(0.67 - \gamma)^{0.4} + 0.56$

where 0.01 and 0.67 are about the minimum and, respectively, the maximum of γ-values, and 0.56 is the minimum of β-values.

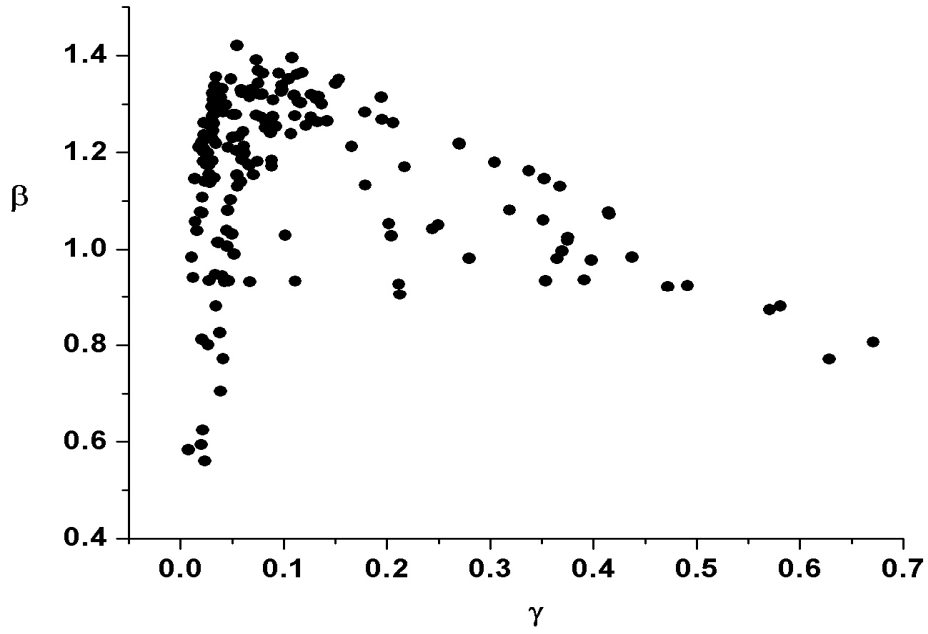


Figure 4.9. Bi-angular <*γ,β*> text classification

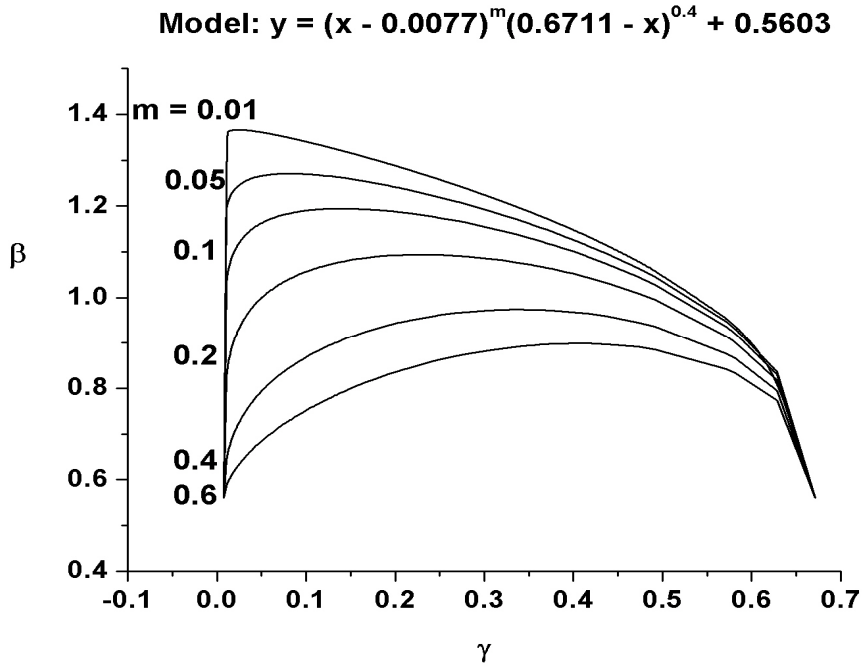


Figure 4.10. The area of <*γ,β*> classification

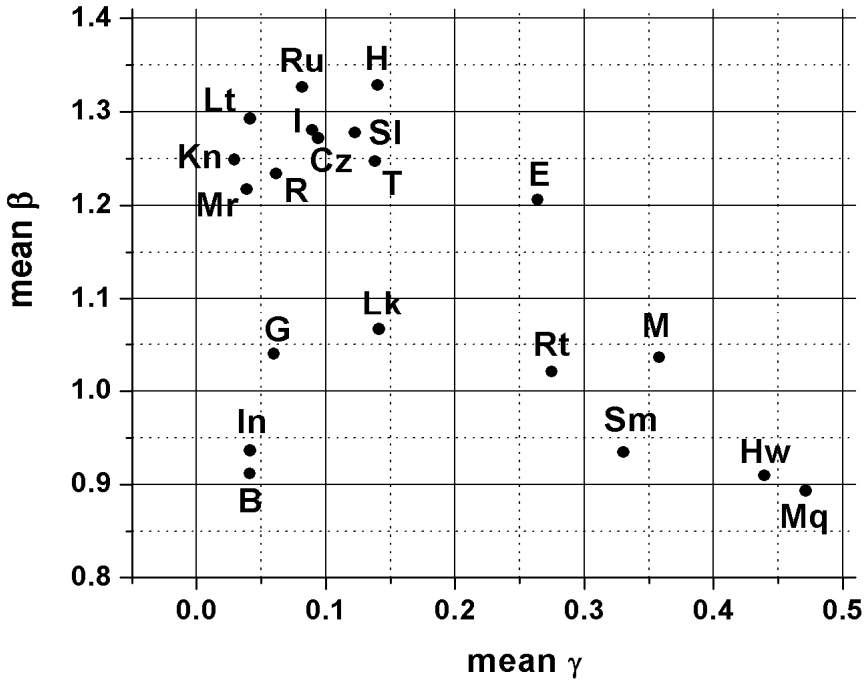


Figure 4.11. Bi-angular <mean γ, mean β,> language classification

From the preceding graphs we have a quite general bi-dimensional picture and classification of texts and languages in terms of the view angles α, β, γ associated to the *h*-point of rank-frequency distributions of word forms. Generally, these angles in radians are limited, for α in the second quadrant ($\pi/2$, π), seemingly with the golden number 1.618… radians as the lower limit, and for β and γ in the first quadrant (0, $\pi/2$), see Figure 4.2 and the following ones. In addition, the triangle angle sum rule

$$\alpha + \beta + \gamma = \pi$$

always holds true.

The same angle data, presented above in binary plots, can be presented more compactly in ternary plots, similar to those currently used in colour science for colour triangle plots, see http://en.wikipedia.org/wiki/CIE_1931_color_space. For this purpose, we shall proceed to the linear rescaling transformation

$$X = (\alpha - \alpha_{min})/(\alpha_{max} - \alpha_{min})$$
$$Y = (\beta - \beta_{min})/(\beta_{max} - \beta_{min})$$
$$Z = (\gamma - \gamma_{min})/(\gamma_{max} - \gamma_{min})$$

where the (min, max) limits confine the *X*, *Y*, *Z* variables in the interval (0, 1). For the considered sample of 176 texts of Table 4.1 we have

$$\alpha_{min} = 1.6327;\ \alpha_{max} = 2.5576$$
$$\beta_{min} = 05603;\ \beta_{max} = 1.4205$$
$$\gamma_{min} = 0.0077;\ \gamma_{max} = 0.6711$$

Finally, the most economic use of ternary plots requires a further transformation as

$$x = X/(X + Y + Z)$$
$$y = Y/(X + Y + Z)$$
$$z = Z/(X + Y + Z)$$

which, obviously, means a re-normalization of the new variables *x*, *y*, *z* at

$$x + y + z = 1$$

Let us give an example of computing for the word-frequency distribution of Goethe's Erlkönig (G 17). Thus, from Table 4.1 we have the angles $\alpha = 2.3985$, $\beta = 0.7043$, $\gamma = 0.0388$ and from the above transformations we get $X = 0.8280$, $Y = 0.1674$, $Z = 0.0468$, and the corresponding normalized triplet $x = 0.7945$, $y = 0.1606$, $z = 0.0449$ Table 4.3 and Table 4.4 illustrate the data presentation in the new canonic variables and the corresponding ternary plot classification of 176 texts, in Figure 4.13, and of 20 languages, in Figure 4.14.

Table 4.3
Linear rescaling (*X*, *Y*, *Z*) and normalizing to $x + y + z = 1$ of Table 4.1 data

Text ID	*N*	*X*	*Y*	*Z*	*x*	*y*	*z*
B 01	761	0.2731	0.7226	0.1005	0.2492	0.6592	0.0917
B 02	352	1.0000	0.0000	0.0241	0.9765	0.0000	0.0235
B 03	515	0.9670	0.0395	0.0190	0.9430	0.0385	0.0185
B 04	483	0.4943	0.5191	0.0561	0.4622	0.4854	0.0524
B 05	406	0.4624	0.5476	0.0635	0.4308	0.5101	0.0592
B 06	687	0.3868	0.6305	0.0616	0.3585	0.5844	0.0571
B 07	557	0.5698	0.4512	0.0389	0.5376	0.4257	0.0367
B 08	268	0.9331	0.0743	0.0210	0.9073	0.0723	0.0204
B 09	550	0.6414	0.3730	0.0404	0.6080	0.3536	0.0383
B 10	556	0.2847	0.7274	0.0782	0.2611	0.6671	0.0718
Cz 01	1044	0.1218	0.8828	0.1037	0.1099	0.7965	0.0936
Cz 02	984	0.1898	0.8029	0.1125	0.1717	0.7265	0.1018
Cz 03	2858	0.0675	0.8827	0.1797	0.0597	0.7812	0.1590

Cz 04	522	0.2687	0.7422	0.0812	0.2460	0.6796	0.0744
Cz 05	999	0.0638	0.8786	0.1901	0.0563	0.7758	0.1679
Cz 06	1612	0.0875	0.8808	0.1542	0.0780	0.7847	0.1374
Cz 07	2014	0.0776	0.8604	0.1945	0.0685	0.7598	0.1718
Cz 08	677	0.2724	0.7416	0.0770	0.2497	0.6797	0.0706
Cz 09	460	0.1765	0.8058	0.1273	0.1591	0.7262	0.1147
Cz 10	1156	0.2217	0.7937	0.0801	0.2024	0.7245	0.0731
E 01	2330	0.0972	0.8632	0.1636	0.0865	0.7680	0.1456
E 02	2971	0.1103	0.8193	0.2022	0.0975	0.7239	0.1786
E 03	3247	0.0453	0.8147	0.2987	0.0391	0.7031	0.2578
E 04	4622	0.0220	0.7652	0.3955	0.0186	0.6470	0.3344
E 05	4760	0.0509	0.8404	0.2576	0.0443	0.7315	0.2242
E 06	4862	0.0116	0.6805	0.5197	0.0096	0.5616	0.4288
E 07	5004	0.0715	0.8730	0.1867	0.0632	0.7718	0.1650
E 08	5083	0.0226	0.5955	0.6146	0.0183	0.4831	0.4985
E 09	5701	0.0491	0.8228	0.2829	0.0425	0.7125	0.2449
E 10	6246	0.0117	0.6627	0.5426	0.0096	0.5445	0.4459
E 11	8193	0.0103	0.6994	0.4971	0.0085	0.5795	0.4119
E 12	9088	0.0271	0.7198	0.4473	0.0227	0.6028	0.3746
E 13	11265	0.0183	0.6012	0.6132	0.0148	0.4877	0.4975
G 01	1095	0.1222	0.8173	0.1882	0.1084	0.7247	0.1669
G 02	845	0.1764	0.7886	0.1498	0.1582	0.7074	0.1344
G 03	500	0.2560	0.7251	0.1213	0.2322	0.6578	0.1100
G 04	545	0.2689	0.7109	0.1216	0.2441	0.6455	0.1104
G 05	559	0.2895	0.7150	0.0876	0.2651	0.6547	0.0802
G 06	545	0.2899	0.7131	0.0894	0.2654	0.6528	0.0818
G 07	263	0.3073	0.6903	0.0948	0.2813	0.6319	0.0868
G 08	965	0.3257	0.6892	0.0707	0.3000	0.6349	0.0651
G 09	653	0.3500	0.6626	0.0712	0.3229	0.6114	0.0657
G 10	480	0.4950	0.5284	0.0430	0.4642	0.4955	0.0403
G 11	468	0.4953	0.5275	0.0437	0.4644	0.4946	0.0410
G 12	251	0.5702	0.4356	0.0586	0.5357	0.4092	0.0550
G 13	460	0.5767	0.4330	0.0528	0.5428	0.4075	0.0497
G 14	184	0.6975	0.3087	0.0456	0.6631	0.2935	0.0434
G 15	593	0.7307	0.2928	0.0198	0.7003	0.2807	0.0190
G 16	518	0.7369	0.2792	0.0289	0.7052	0.2672	0.0276
G 17	225	0.8280	0.1674	0.0468	0.7945	0.1606	0.0449
H 01	2044	0.0000	0.8766	0.2817	0.0000	0.7568	0.2432
H 02	1288	0.0047	0.9195	0.2195	0.0041	0.8040	0.1919
H 03	403	0.0180	0.9088	0.2148	0.0157	0.7961	0.1882
H 04	936	0.0376	0.9312	0.1584	0.0334	0.8261	0.1405
H 05	413	0.1575	0.8296	0.1229	0.1419	0.7474	0.1107

Hw 01	282	0.5012	0.4346	0.1561	0.4590	0.3980	0.1429
Hw 02	1829	0.2379	0.4357	0.5217	0.1990	0.3645	0.4365
Hw 03	3507	0.0950	0.4921	0.6479	0.0769	0.3985	0.5246
Hw 04	7892	0.0700	0.3639	0.8488	0.0546	0.2837	0.6617
Hw 05	7620	0.1008	0.4235	0.7286	0.0805	0.3380	0.5815
Hw 06	12356	0.0340	0.2860	1.0000	0.0257	0.2167	0.7576
I 01	11760	0.0543	0.9336	0.1320	0.0485	0.8337	0.1179
I 02	6064	0.0564	0.9203	0.1462	0.0503	0.8195	0.1302
I 03	854	0.1322	0.8316	0.1556	0.1181	0.7429	0.1390
I 04	3258	0.1707	0.8286	0.1059	0.1545	0.7498	0.0958
I 05	1129	0.3368	0.6731	0.0759	0.3102	0.6199	0.0699
In 01	376	0.4595	0.5566	0.0560	0.4286	0.5192	0.0522
In 02	373	0.5051	0.4994	0.0666	0.4716	0.4663	0.0621
In 03	347	0.5658	0.4472	0.0497	0.5324	0.4208	0.0468
In 04	343	0.5896	0.4364	0.0305	0.5581	0.4131	0.0288
In 05	414	0.7523	0.2460	0.0505	0.7173	0.2346	0.0481
Kn 003	3188	0.1502	0.9026	0.0385	0.1376	0.8271	0.0353
Kn 004	1050	0.3301	0.7221	0.0218	0.3073	0.6723	0.0203
Kn 005	4869	0.1285	0.9247	0.0401	0.1175	0.8458	0.0366
Kn 006	5231	0.3074	0.7465	0.0218	0.2858	0.6940	0.0202
Kn 011	4541	0.3019	0.7569	0.0159	0.2809	0.7043	0.0148
Kn 012	4141	0.4113	0.6361	0.0202	0.3853	0.5958	0.0189
Kn 013	1302	0.3189	0.7240	0.0349	0.2959	0.6717	0.0324
Kn 016	4735	0.1819	0.8702	0.0363	0.1672	0.7995	0.0333
Kn 017	4316	0.1170	0.9201	0.0621	0.1064	0.8371	0.0565
Lk 01	345	0.5496	0.4337	0.0898	0.5122	0.4042	0.0836
Lk 02	1633	0.1310	0.7094	0.3159	0.1133	0.6135	0.2732
Lk 03	809	0.2129	0.6656	0.2584	0.1873	0.5854	0.2273
Lk 04	219	0.4090	0.5452	0.1412	0.3734	0.4977	0.1289
Lt 01	3311	0.0365	1.0000	0.0708	0.0330	0.9031	0.0639
Lt 02	4010	0.0475	0.9662	0.0992	0.0427	0.8682	0.0891
Lt 03	4931	0.1686	0.8855	0.0352	0.1548	0.8130	0.0323
Lt 04	4285	0.1991	0.8407	0.0506	0.1826	0.7710	0.0464
Lt 05	1354	0.2366	0.8164	0.0298	0.2185	0.7539	0.0275
Lt 06	829	0.4451	0.6013	0.0180	0.4182	0.5649	0.0169
M 01	2062	0.1178	0.6055	0.4689	0.0988	0.5079	0.3933
M 02	1175	0.1252	0.5323	0.5535	0.1034	0.4395	0.4570
M 03	1434	0.1543	0.5070	0.5458	0.1278	0.4200	0.4522
M 04	1289	0.1052	0.5812	0.5180	0.0874	0.4825	0.4301
M 05	3620	0.1177	0.5400	0.5539	0.0972	0.4457	0.4571
Mq 01	2330	0.1186	0.2447	0.9357	0.0913	0.1884	0.7203
Mq 02	457	0.2989	0.5438	0.2964	0.2624	0.4774	0.2602

Mq 03	1509	0.0508	0.3728	0.8641	0.0395	0.2895	0.6711
Mr 001	2998	0.1692	0.8753	0.0474	0.1550	0.8016	0.0434
Mr 002	2922	0.2729	0.7557	0.0579	0.2512	0.6955	0.0533
Mr 003	4140	0.3526	0.6916	0.0300	0.3282	0.6438	0.0279
Mr 004	6304	0.0289	0.9350	0.1656	0.0256	0.8278	0.1467
Mr 005	4957	0.0694	0.9408	0.1017	0.0624	0.8461	0.0915
Mr 006	3735	0.1369	0.8778	0.0893	0.1240	0.7951	0.0809
Mr 007	3162	0.1950	0.8344	0.0646	0.1783	0.7627	0.0590
Mr 008	5477	0.1199	0.8696	0.1236	0.1077	0.7813	0.1110
Mr 009	6206	0.3307	0.7147	0.0305	0.3073	0.6643	0.0284
Mr 010	5394	0.0974	0.8652	0.1606	0.0867	0.7703	0.1430
Mr 015	4693	0.1305	0.8940	0.0771	0.1184	0.8115	0.0700
Mr 016	3642	0.3337	0.7154	0.0253	0.3106	0.6658	0.0236
Mr 017	4170	0.3316	0.7162	0.0273	0.3084	0.6662	0.0254
Mr 018	4062	0.1365	0.8874	0.0775	0.1239	0.8058	0.0703
Mr 020	3943	0.3725	0.6750	0.0238	0.3477	0.6301	0.0222
Mr 021	3846	0.4460	0.5988	0.0202	0.4188	0.5623	0.0189
Mr 022	4099	0.1230	0.8882	0.0951	0.1112	0.8029	0.0859
Mr 023	4142	0.3229	0.7234	0.0301	0.3000	0.6720	0.0280
Mr 024	4255	0.2767	0.7652	0.0404	0.2556	0.7070	0.0374
Mr 026	4146	0.2348	0.8133	0.0363	0.2165	0.7500	0.0335
Mr 027	4128	0.2456	0.7800	0.0644	0.2253	0.7156	0.0591
Mr 028	5191	0.3064	0.7427	0.0281	0.2844	0.6895	0.0260
Mr 029	3424	0.9926	0.0266	0.0001	0.9740	0.0261	0.0001
Mr 030	5504	0.2432	0.8149	0.0225	0.2251	0.7541	0.0208
Mr 031	5105	0.2423	0.8112	0.0286	0.2239	0.7496	0.0264
Mr 032	5195	0.2517	0.7957	0.0357	0.2324	0.7347	0.0330
Mr 033	4339	0.3022	0.7506	0.0236	0.2808	0.6973	0.0220
Mr 034	3489	0.5996	0.4438	0.0069	0.5709	0.4226	0.0065
Mr 035	1862	0.4911	0.5560	0.0126	0.4634	0.5246	0.0119
Mr 036	4205	0.1908	0.8544	0.0443	0.1752	0.7842	0.0407
Mr 038	4078	0.3695	0.6723	0.0313	0.3443	0.6265	0.0292
Mr 040	5218	0.2885	0.7684	0.0198	0.2680	0.7137	0.0184
Mr 043	3356	0.4731	0.5775	0.0098	0.4462	0.5446	0.0093
Mr 046	4186	0.3549	0.6831	0.0377	0.3299	0.6350	0.0351
Mr 052	3549	0.1802	0.8577	0.0549	0.1649	0.7849	0.0502
Mr 149	2946	0.2701	0.7861	0.0224	0.2504	0.7288	0.0208
Mr 150	3372	0.2713	0.7745	0.0357	0.2508	0.7161	0.0331
Mr 151	4843	0.0872	0.8942	0.1372	0.0779	0.7994	0.1226
Mr 154	3601	0.2719	0.7760	0.0330	0.2516	0.7179	0.0305
Mr 288	4060	0.1949	0.8558	0.0369	0.1792	0.7869	0.0339
Mr 289	4831	0.1483	0.8967	0.0489	0.1355	0.8197	0.0447

Mr 290	4025	0.5561	0.4923	0.0046	0.5281	0.4675	0.0043
Mr 291	3954	0.2074	0.8394	0.0406	0.1907	0.7719	0.0374
Mr 292	4765	0.2179	0.8320	0.0356	0.2007	0.7665	0.0328
Mr 293	3337	0.3773	0.6808	0.0094	0.3535	0.6377	0.0088
Mr 294	3825	0.1920	0.8555	0.0414	0.1763	0.7857	0.0380
Mr 295	4895	0.2036	0.8454	0.0383	0.1872	0.7776	0.0352
Mr 296	3836	0.1866	0.8585	0.0449	0.1712	0.7876	0.0412
Mr 297	4605	0.1988	0.8534	0.0346	0.1829	0.7852	0.0319
R 01	1738	0.2360	0.7832	0.0738	0.2159	0.7166	0.0675
R 02	2279	0.1181	0.8835	0.1080	0.1064	0.7962	0.0974
R 03	1264	0.1711	0.8335	0.0990	0.1551	0.7552	0.0897
R 04	1284	0.1918	0.8345	0.0688	0.1752	0.7620	0.0628
R 05	1032	0.2534	0.7590	0.0809	0.2318	0.6942	0.0740
R 06	695	0.4133	0.6050	0.0575	0.3842	0.5624	0.0535
Rt 01	968	0.1443	0.4850	0.5882	0.1185	0.3984	0.4831
Rt 02	845	0.2258	0.5701	0.3643	0.1946	0.4914	0.3140
Rt 03	892	0.2406	0.5606	0.3560	0.2079	0.4844	0.3076
Rt 04	625	0.2748	0.5726	0.2927	0.2410	0.5023	0.2567
Rt 05	1059	0.2680	0.4894	0.4101	0.2295	0.4192	0.3513
Ru 01	753	0.2708	0.7486	0.0701	0.2485	0.6871	0.0644
Ru 02	2595	0.0786	0.9045	0.1359	0.0702	0.8083	0.1215
Ru 03	3853	0.1196	0.8947	0.0915	0.1081	0.8092	0.0827
Ru 04	6025	0.0706	0.9341	0.1086	0.0634	0.8390	0.0976
Ru 05	17205	0.0057	0.9709	0.1514	0.0050	0.8607	0.1342
Sl 01	756	0.1767	0.8166	0.1130	0.1597	0.7381	0.1022
Sl 02	1371	0.1958	0.7908	0.1199	0.1770	0.7147	0.1083
Sl 03	1966	0.0925	0.8899	0.1355	0.0827	0.7960	0.1212
Sl 04	3491	0.0234	0.7635	0.3957	0.0198	0.6456	0.3346
Sl 05	5588	0.0983	0.9096	0.1019	0.0886	0.8196	0.0918
Sm 01	1487	0.1235	0.4212	0.7000	0.0992	0.3384	0.5624
Sm 02	1171	0.1767	0.4886	0.5384	0.1468	0.4059	0.4473
Sm 03	617	0.4228	0.4013	0.3085	0.3733	0.3543	0.2724
Sm 04	736	0.1958	0.4376	0.5778	0.1617	0.3613	0.4771
Sm 05	447	0.3994	0.4276	0.3070	0.3522	0.3771	0.2707
T 01	1551	0.1422	0.8086	0.1715	0.1267	0.7205	0.1528
T 02	1827	0.1178	0.8290	0.1791	0.1046	0.7363	0.1591
T 03	2054	0.1411	0.7581	0.2386	0.1240	0.6663	0.2097

The variables can be plotted together using the principle of the ternary plot presented in Figure 4.12. The three outside arrows indicate the direction to look when fixing the corresponding coordinates. The positioning of actual texts is shown in Figure 4.13.

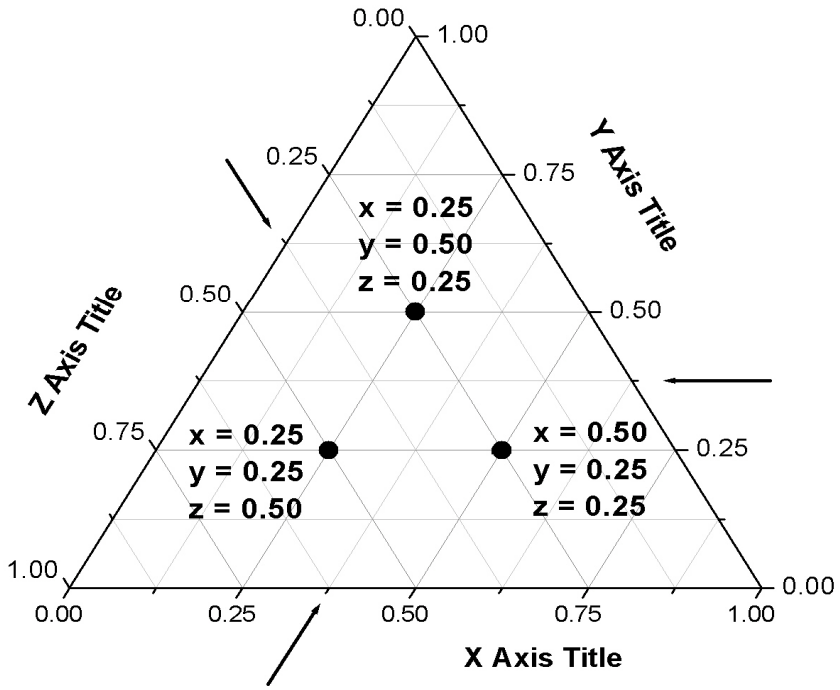


Figure 4.12. Ternary plot

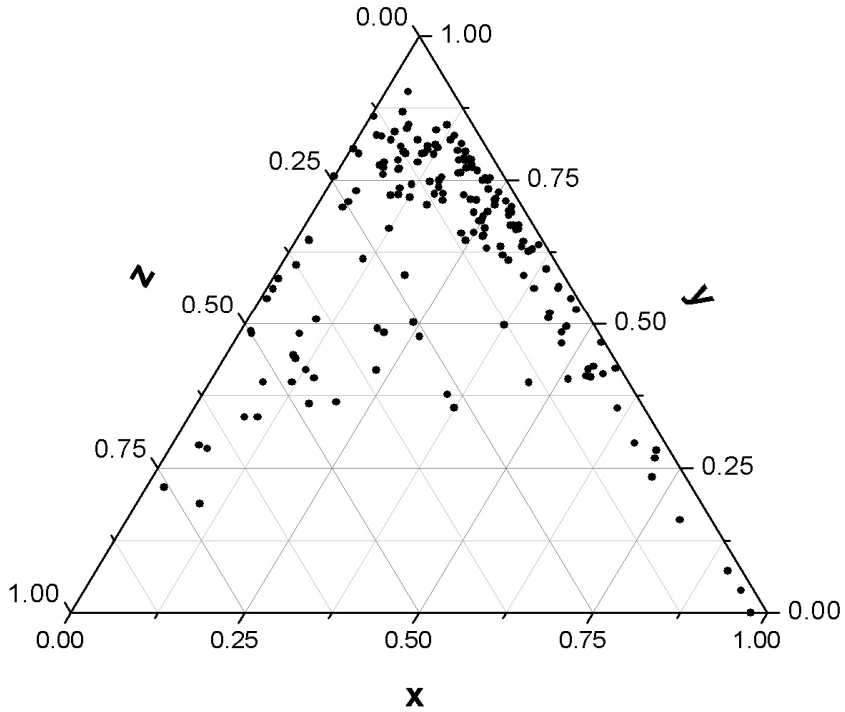


Figure 4.13. Ternary plot (x, y, z) text classification normalized to $x + y + z = 1$

As can be seen, the points fill only a part of the figure but this part seems to be quite clearly demarcated. Quite possibly it builds a strange attractor whose exact form is not yet known. If our conjecture is correct then "abnormal" texts (e.g. dadaistic texts or texts with a frequent refrain) can be mechanically recognized by their position in the ternary plot. For example a short jodling text

Hola dere tütü, hola dere tütü,
Hola dere tütü, hola dero.

with N = 11, V = 4, $f(1)$ = 4, and h =3 and, correspondingly with α = 2.4981, β = 0.3218, γ = 0.3218 would occupy the position which is very distant from the attractor, as it can be directly seen in the bi-angular graphs of Figures 4.3, 4.6, and 4.9. It would be interesting to examine the positioning of musical compositions and to examine how far they are from language texts.

If we take averages from all texts in a language, we obtain the result presented in Table 4.4. This presentation is suitable for language typology; however, it will not be easy to find and combine all factors contributing to a given position.

Table 4.4
Ternary average coordinates of 20 languages

Language	mean x	mean y	mean z
B Bulgarian	0.5734	0.3796	0.0470
Cz Czech	0.1401	0.7435	0.1164
E English	0.0366	0.6398	0.3237
G German	0.4146	0.5135	0.0719
H Hungarian	0.0390	0.7861	0.1749
Hw Hawaiian	0.1493	0.3332	0.5175
I Italian	0.1363	0.7531	0.1106
In Indonesian	0.5416	0.4108	0.0476
Kn Kannada	0.2315	0.7386	0.0298
Lk Lakota	0.2965	0.5252	0.1782
Lt Latin	0.1750	0.7790	0.0460
M Maori	0.1029	0.4591	0.4379
Mq Marquesan	0.1310	0.3184	0.5505
Mr Marathi	0.2547	0.7023	0.0429
R Romanian	0.2114	0.7144	0.0741
Rt Rarotongan	0.1983	0.4591	0.3426
Ru Russian	0.0991	0.8009	0.1001
Sl Slovenian	0.1056	0.7428	0.1516
Sm Samoan	0.2266	0.3674	0.4060
T Tagalog	0.1184	0.7077	0.1739

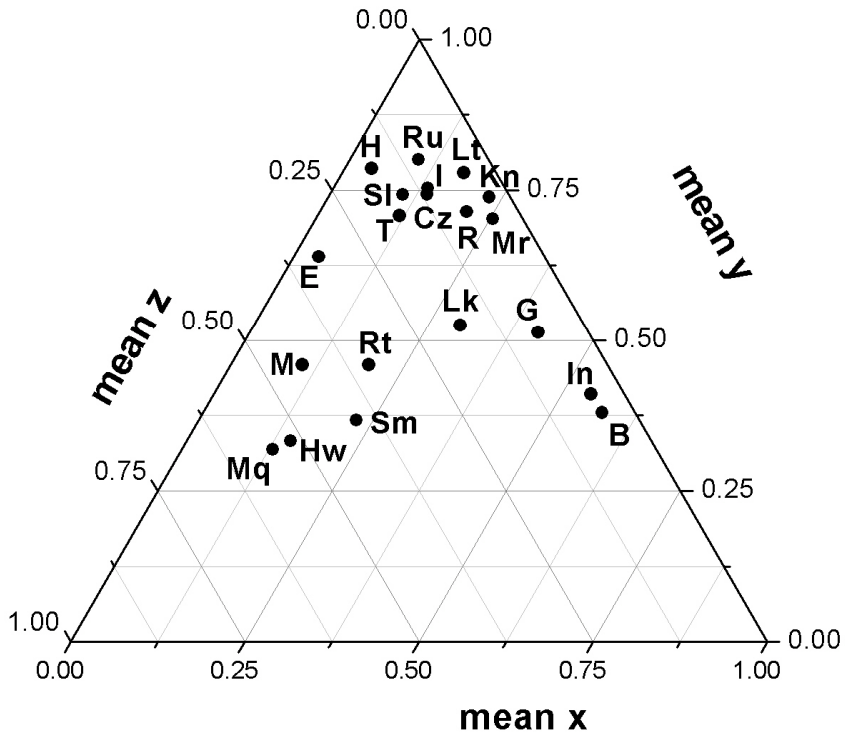


Figure 4.14. Ternary plot (mean x, mean y, mean z) language classification normalized to $x + y + z = 1$

As can be seen both in Figure 4.13 and 4.14 the variability of texts and of languages is rather restricted. Extremely isolating languages have their small domain ($x < 0.25$, $y < 0.5$, $z > 0.25$); strong increase of synthetism places the languages in a very dense cluster ($x < 0.25$, $y > 0.70$, $z < 0.22$); four languages (Lakota, German, Bulgarian and Indonesian) contain a factor which is unknown. Perhaps a more thorough analysis of texts in these languages and the study of their grammar would allow us to trace up their attribute space. We conjecture that a slight change in the way of writing words in these languages would bring a solution. Nevertheless, the points in all of the figures represent some strange attractors which will be studied in the future.

Thus the *h*-point opens a view to some not unexamined aspects of self-regulation, morphological classification of languages, and the rise of strange attractors in linguistics, and joins text analysis with the golden section. The latter aspect is further scrutinized in Tuzzi, Popescu, Altmann (2009).

5. Arc length

5.1. Arc length and associated typological indicators

Out of many possibilities to characterize a rank-frequency sequence is the use of arc length along the ranked frequencies. For continuous functions one uses the appropriate integral, for an empirical ranked sequence one sums the lengths of straight lines joining two neighbouring frequencies. The Euclidian distance between frequency $f(r)$ and $f(r+1)$ is defined as

$$D_r = [(f(r) - f(r+1))^2 + 1]^{1/2},$$

and shown in Figure 5.1.

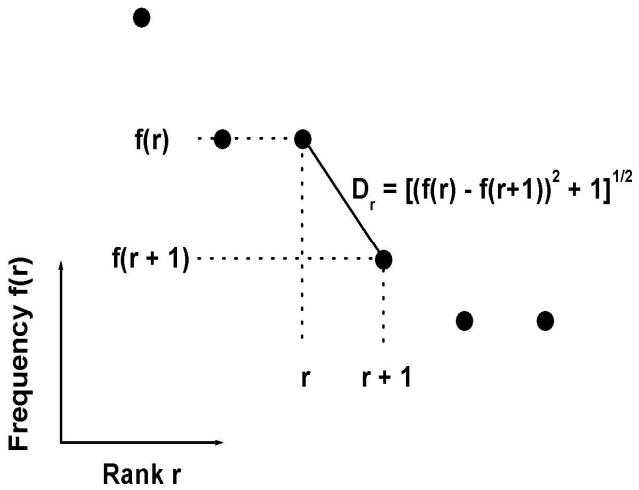


Figure 5.1. Length of one arc segment

Hence the arc length is defined as

$$(5.1) \quad L = \sum_{r=1}^{V-1} D_r = \sum_{r=1}^{V-1} [(f(r) - f(r+1))^2 + 1]^{1/2}.$$

As an example consider the last column in Table 2.2, Chapter 2, yielding

r	1, 2, 3, 4, 5
$f(r)$	4, 2, 1, 1, 1

The arc length is

$$[(4-2)^2+1]^{1/2}+[(2-1)^2+1]^{1/2}+[(1-1)^2+1]^{1/2}+[(1-1)^2+1]^{1/2}=5.650.$$

However, arc length itself is not sufficient for characterization because it depends on text length N whose simplest representative is the greatest frequency $f(1)$, and on the vocabulary (or inventory of forms) level the size V. In order to warrant comparability, the authors proposed four different indicators which may be used to characterize the rank-frequency sequence (Popescu, Mačutek, Altmann 2008). They are as follows

(5.2) $$B_1=\frac{L}{L_{\max}},$$

where

$$L_{\max} = [(f(1) - 1)^2+1]^{1/2}+V-2$$

representing the case when $f(1) > 1$ and all other $f(r) = 1$. This is a simply relativized arc length. Another normalization can be performed by defining

(5.3) $$B_2=\frac{L-L_{\min}}{L_{\max}-L_{\min}},$$

where

$$L_{\min} = [(V-1)^2+(f(1)-1)^2]^{1/2}$$

represents the length of the straight line joining the points P_1 (V, 1) and P_2 (1, $f(1)$), see Figure 4.1 of Chapter 4. For the above example we would obtain

$$L_{\max} = [(4-1)^2+1]^{1/2}+5-2=6.162$$
$$L_{\min} = [(5-1)^2+(4-1)^2]^{1/2} = 5.000.$$

Hence

$$B_1 = 5.650/6.162 = 0.917$$
$$B_2 = (5.650-5.000)/(6.162-5.000) = 0.559.$$

Both indicators lie in the interval <0, 1>. Using the individual components of L_{min} (that is $(V-1)$ or $(f(1)-1)$) one can set up two further indicators, namely

(5.4) $$B_3=\frac{V-1}{L}$$

and

(5.5) $$B_4 = \frac{f(1)-1}{L}.$$

Since L is always greater than V-1 and $f(1)$-1, these last indicators lie in the interval <0,1>, too. For our example we have $B_3 = \frac{5-1}{5.650} = 0.708$ and $B_4 = \frac{4-1}{5.650} = 0.531$.

In order to be able to test differences between particular indicators, we derive estimations of their variances. We use the well known delta method (cf. Oehlert 1992) based on a linear approximation of a non-linear function. V and $f(1)$ are considered to be fixed. In the following, $f_1,\ldots f_V$ are random variables (i.e., functions, not numbers, which are denoted $f(1),\ldots,f(V)$).

L is a function of frequencies, i.e., $L = L(f_1,\ldots,f_V) = \sum_{r=1}^{V-1}\left[(f_r - f_{r+1})^2 + 1\right]^{\frac{1}{2}}$.

Approximating the function by its first order Taylor polynomial (f_f is a constant, hence its derivative is zero), we obtain

(5.6) $$L(f_1,\ldots,f_V) \cong L(\theta_1,\ldots,\theta_V) + (f_2-\theta_2)\left.\frac{\partial L}{\partial f_2}\right|_\theta + \ldots + (f_V-\theta_V)\left.\frac{\partial L}{\partial f_V}\right|_\theta,$$

where $\theta = (\theta_1,\ldots\ \theta_V)$ is the mean vector of $(f_1,\ldots,f_V)$ (i.e., $E(f_1) = \theta_1,\ldots,E(f_V) = \theta_V$). Denote

(5.7) $$a_r = \left.\frac{\partial L}{\partial f_r}\right|_\theta = -\frac{\theta_{r-1}-\theta_r}{\sqrt{(\theta_{r-1}-\theta_r)^2+1}} + \frac{\theta_r-\theta_{r+1}}{\sqrt{(\theta_r-\theta_{r+1})^2+1}},\ r = 2,3,\ldots,V-1,$$

(5.8) $$a_V = \left.\frac{\partial L}{\partial f_V}\right|_\theta = -\frac{\theta_{V-1}-\theta_V}{\sqrt{(\theta_{V-1}-\theta_V)^2+1}}.$$

Inserting the derivatives into (5.6) we obtain

$$L(f_1,\ldots,f_V) \cong L(\theta_1,\ldots,\theta_V) + \sum_{k=2}^{V} a_k(f_k-\theta_k)$$

and an approximation of the variance

(5.9) $$Var(L) \cong \sum_{r=2}^{V} a_r^2 Var(f_r) + 2\sum_{r=2}^{V-1}\sum_{s=r+1}^{V} a_r a_s Cov(f_r, f_s).$$

Variances and covariances from the last formula can be estimated from the properties of the multinomial distribution. We have N words in V categories with the fixed number of words in the first category, resulting in the conditional probability

(5.10)
$$P\left(f_2 = f(2),\ldots, f_V = f(V) \middle| f_1 = f(1)\right) = \frac{(N - f(1))!}{f(2)!\ldots f(V)!}\left(\frac{p_2}{1-p_1}\right)^{f(2)} \ldots \left(\frac{p_V}{1-p_V}\right)^{f(V)}$$

which means that the conditional distribution is again multinomial having now the parameters $N - f(1), \frac{p_2}{1-p_1},\ldots,\frac{p_V}{1-p_1}$, which means that for $r = 2,\ldots,V$ we have

(5.11) $E(f_r) = (N - f(1))\frac{p_r}{1-p_1}$ (note that $E(f_r) = \theta_r$),

(5.12) $Var(f_r) = (N - f(1))\frac{p_r}{1-p_1}\left(1 - \frac{p_r}{1-p_1}\right)$,

(5.13) $Cov(f_r, f_s) = -(N - f(1))\frac{p_r p_s}{(1-p_1)^2}$.

As $f(1)$ is a constant, it holds $E(f(1)) = f(1)$. The values of N and $f(1)$ are known, the others can be estimated as $\hat{p}_k = \frac{f(k)}{N}$, $k = 1,\ldots,V$, thus making possible to estimate also

(5.14) $$\hat{a}_r = -\frac{(N - f(1))\left(\frac{\hat{p}_{r-1} - \hat{p}_r}{1-\hat{p}_1}\right)}{\sqrt{(N - f(1))^2\left(\frac{\hat{p}_{r-1} - \hat{p}_r}{1-\hat{p}_1}\right)^2 + 1}} + \frac{(N - f(1))\left(\frac{\hat{p}_r - \hat{p}_{r+1}}{1-\hat{p}_1}\right)}{\sqrt{(N - f(1))^2\left(\frac{\hat{p}_r - \hat{p}_{r+1}}{1-\hat{p}_1}\right)^2 + 1}},$$

for $r = 2,\ldots,V$-1 and

$$(5.15)\quad \hat{a}_V = -\frac{\left(N - f(1)\right)\left(\dfrac{\hat{p}_{V-1} - \hat{p}_V}{1-\hat{p}_1}\right)}{\sqrt{\left(N - f(1)\right)^2\left(\dfrac{\hat{p}_{V-1} - \hat{p}_V}{1-\hat{p}_1}\right)^2 + 1}}.$$

Finally, we obtain the estimation

$$(5.16)\quad Var(L) = \frac{N - f(1)}{1-\hat{p}_1}\sum_{r=2}^{V}\hat{a}_r^2\hat{p}_r\left(1-\frac{\hat{p}_r}{1-\hat{p}_1}\right) - 2\frac{N - f(1)}{\left(1-\hat{p}_1\right)^2}\sum_{r=2}^{V-1}\sum_{s=r+1}^{V}\hat{a}_r\hat{a}_s\hat{p}_r\hat{p}_s\,.$$

As $L_{\max}$ and $L_{\min}$ depend only on $f(1)$ and V (which are fixed), we have

$$(5.17)\quad Var\left(B_1\right) = Var\left(\frac{L}{L_{\max}}\right) = \frac{Var\left(L\right)}{L_{\max}^2},$$

$$(5.18)\quad Var\left(B_2\right) = Var\left(\frac{L - L_{\min}}{L_{\max} - L_{\min}}\right) = \frac{Var\left(L\right)}{\left(L_{\max} - L_{\min}\right)^2}.$$

Analogously we derive variances for the indicators B_3 and B_4. Denote $M = 1/L$. We apply the delta method again and obtain

$$M\left(f_1,\ldots,f_V\right) \cong M\left(\theta_1,\ldots,\theta_V\right) + \left(f_2 - \theta_2\right)\left.\frac{\partial M}{\partial f_2}\right|_\theta + \ldots + \left(f_V - \theta_V\right)\left.\frac{\partial M}{\partial f_V}\right|_\theta.$$

The partial derivatives are

$$(5.19)\quad b_r = \left.\frac{\partial M}{\partial f_r}\right|_\theta = \frac{1}{\left(\sum_{r=1}^{V-1}\sqrt{\left(\theta_r - \theta_{r+1}\right)^2 + 1}\right)^2}\left(\frac{\theta_{r-1} - \theta_r}{\sqrt{\left(\theta_{r-1} - \theta_r\right)^2 + 1}} - \frac{\theta_r - \theta_{r+1}}{\sqrt{\left(\theta_r - \theta_{r+1}\right)^2 + 1}}\right),$$

where $r = 2,3,\ldots,V\text{-}1$, and

(5.20) $$b_V = \left.\frac{\partial L}{\partial f_V}\right|_\theta = \frac{1}{\left(\sum_{r=1}^{V-1}\sqrt{(\theta_r-\theta_{r+1})^2+1}\right)^2}\frac{\theta_{V-1}-\theta_V}{\sqrt{(\theta_{V-1}-\theta_V)^2+1}}.$$

We have an approximation of the variance

(5.21) $$Var(M) \cong \sum_{r=2}^{V}\hat{b}_r^2 Var(f_r) + 2\sum_{r=2}^{V-1}\sum_{s=r+1}^{V}\hat{b}_r\hat{b}_s Cov(f_r, f_s).$$

The unknown parameters can be estimated in the same way as in the formula for $Var(L)$, i.e.

$$E(f_r) = (N - f(1))\frac{p_r}{1-p_1},$$

$$Var(f_r) = (N - f(1))\frac{p_r}{1-p_1}\left(1-\frac{p_r}{1-p_1}\right),$$

$$Cov(f_r, f_s) = -(N - f(1))\frac{p_r p_s}{(1-p_1)^2},$$

cf. (5.11), (5.12) and (5.13), and

$$\hat{p}_r = \frac{f(r)}{N},\ k = 1,\ldots,V.$$

Thus we obtain the variances

(5.22) $$Var(B_3) = Var\left(\frac{V-1}{L}\right) = (V-1)^2 Var(M),$$

(5.23) $$Var(B_4) = Var\left(\frac{f_1-1}{L}\right) = (f_1-1)^2 Var(M).$$

We demonstrate calculations of particular variances on the example from the beginning of this chapter. Consider the frequencies

r	1, 2, 3, 4, 5
$f(r)$	4, 2, 1, 1, 1,

hence we have $f(1) = 4$, $V = 5$, $N = 9$ and $L_{\max} = 6.162$, $L_{\min} = 5$ (see above for computations of the last two values). As the estimations of unknown probabilities

are $\hat{p}_k = \dfrac{f(k)}{N}$, k = 1,…,V, we obtain $\hat{p}_1 = 0.444$, $\hat{p}_2 = 0.222$, $\hat{p}_3 = \hat{p}_4 = \hat{p}_5$ = 0.111. Then, substituting the values into (5.14) and (5.15) we estimate

$$\hat{a}_2 = -\frac{(9-4)\left(\dfrac{0.444-0.222}{1-0.444}\right)}{\sqrt{(9-4)^2\left(\dfrac{0.444-0.222}{1-0.444}\right)^2+1}} + \frac{(9-4)\left(\dfrac{0.222-0.111}{1-0.444}\right)}{\sqrt{(9-4)^2\left(\dfrac{0.222-0.111}{1-0.444}\right)^2+1}} = -0.187,$$

similarly $\hat{a}_3 = -0.707$, $\hat{a}_4 = \hat{a}_5 = 0$. In this case the computations are simplified because $\hat{p}_3 = \hat{p}_4 = \hat{p}_5$, which results in many terms in the sums being zero. Now, substituting the values of $\hat{a}_2,\ldots,\hat{a}_5$ into (5.16) we have

$$Var(L) = \frac{9-4}{1-0.444}\left[(-0.187)^2 \times 0.222 \times \left(1-\frac{0.222}{1-0.444}\right) + (-0.707)^2 \times 0.111 \times \left(1-\frac{0.111}{1-0.444}\right)\right]$$
$$-2\frac{9-4}{(1-0.444)^2} \times (-0.187) \times (-0.707) \times 0.222 \times 0.111 = 0.336,$$

and, according to (5.17) and (5.18),

$$Var(B_1) = \frac{0.336}{6.162^2} = 0.009,$$

$$Var(B_2) = \frac{0.336}{(6.162-5)^2} = 0.249.$$

Obviously, for r = 2,3,…,V it holds $b_r = \dfrac{-a_r}{\left(\sum_{r=1}^{V-1}\sqrt{(\theta_r-\theta_{r+1})^2+1}\right)^2}$, hence we have the estimation

$$Var(M) = \left[\sqrt{(4-1.996)^2+1} + \sqrt{(1.996-0.998)^2+1}\right]^{-2} \times$$
$$\times \frac{9-4}{1-0.444}\left[0.187^2 \times 0.222 \times \left(1-\frac{0.222}{1-0.444}\right) + 0.707^2 \times 0.111 \times \left(1-\frac{0.111}{1-0.444}\right)\right] -$$
$$-2\left[\sqrt{(4-1.996)^2+1} + \sqrt{(1.996-0.998)^2+1}\right]^{-4} \times \frac{9-4}{(1-0.444)^2} \times$$
$$\times 0.187 \times 0.707 \times 0.222 \times 0.111 = 0.033$$

and according to (5.22) and (5.23) we obtain

$$Var(B_3) = 4^2 \times 0.033 = 0.534,$$
$$Var(B_4) = 3^2 \times 0.033 = 0.300.$$

Consider another example,

r	1, 2, 3, 4, 5, 6, 7
$f(r)$	8, 5, 2, 2, 1, 1, 1

For these frequencies we obtain the values $L^* = 10.739$, $L^*_{\max} = 12.071$, $L^*_{\min} = 9.220$, $B_1^* = 0.890$, $B_2^* = 0.533$, $B_3^* = 0.559$, $B_4^* = 0.652$ and variances $Var\left(B_1^*\right) = 0.014$, $Var\left(B_2^*\right) = 0.254$, $Var\left(B_3^*\right) = 0.001$ and $Var\left(B_4^*\right) = 0.002$. Now we can test the differences between corresponding indicators. We evaluate the expressions

$$\frac{B_1 - B_1^*}{\sqrt{Var(B_1) + Var(B_1^*)}} = \frac{0.917 - 0.890}{\sqrt{0.009 + 0.014}} = 0.178,$$
$$\frac{B_2 - B_2^*}{\sqrt{Var(B_2) + Var(B_2^*)}} = \frac{0.559 - 0.533}{\sqrt{0.249 + 0.254}} = 0.037,$$
$$\frac{B_3 - B_3^*}{\sqrt{Var(B_3) + Var(B_3^*)}} = \frac{0.708 - 0.559}{\sqrt{0.534 + 0.001}} = 0.204,$$
$$\frac{B_4 - B_4^*}{\sqrt{Var(B_4) + Var(B_4^*)}} = \frac{0.531 - 0.652}{\sqrt{0.300 + 0.002}} = -0.171,$$

which means that in all four cases the difference is not significant.

Let us consider again 100 texts in 20 languages and compute these indicators. The results are presented in Table 5.1 (from Popescu, Mačutek, Altmann 2008).

Table 5.1
Indicators B_i of 100 texts in 20 languages
(B = Bulgarian, Cz = Czech, E = English, G = German, H = Hungarian, Hw = Hawaiian, I = Italian, In = Indonesian, Kn = Kannada, Lk = Lakota, Lt = Latin, M = Maori, Mq = Marquesan, Mr = Marathi, R = Romanian, Rt = Rarotongan, Ru = Russian, Sl = Slovenian, Sm = Samoan, T = Tagalog)

ID	V	$f(1)$	L	B_1	B_2	B_3	B_4
B 01	400	40	428.45	0.980	0.763	0.931	0.091
B 02	201	13	205.38	0.973	0.470	0.974	0.058
B 03	285	15	289.80	0.976	0.430	0.980	0.048
B 04	286	21	297.03	0.977	0.618	0.959	0.067
B 05	238	19	247.30	0.974	0.588	0.958	0.073
Cz 01	638	58	684.17	0.987	0.835	0.931	0.083
Cz 02	543	56	586.22	0.984	0.809	0.925	0.094
Cz 03	1274	182	1432.06	0.986	0.875	0.889	0.126
Cz 04	323	27	341.99	0.986	0.790	0.942	0.076
Cz 05	556	84	626.98	0.984	0.868	0.885	0.132
E 01	939	126	1042.85	0.982	0.834	0.899	0.120
E 02	1017	168	1157.22	0.979	0.837	0.878	0.144
E 03	1001	229	1204.91	0.982	0.890	0.830	0.189
E 04	1232	366	1567.31	0.983	0.911	0.785	0.233
E 05	1495	297	1760.86	0.984	0.894	0.848	0.168
E 07	1597	237	1800.70	0.983	0.861	0.886	0.131
E 13	1659	780	2388.47	0.980	0.921	0.694	0.326
G 05	332	30	351.41	0.979	0.716	0.942	0.083
G 09	379	30	398.43	0.981	0.718	0.949	0.073
G 10	301	18	309.84	0.980	0.602	0.968	0.055
G 11	297	18	306.80	0.983	0.664	0.965	0.055
G 12	169	14	175.44	0.974	0.601	0.958	0.074
G 14	129	10	132.54	0.974	0.546	0.966	0.068
G 17	124	11	127.96	0.969	0.527	0.961	0.078
H 01	1079	225	1288.83	0.991	0.939	0.836	0.174
H 02	789	130	907.18	0.990	0.925	0.869	0.142
H 03	291	48	332.44	0.989	0.915	0.872	0.141
H 04	609	76	674.06	0.988	0.885	0.902	0.111
H 05	290	32	314.40	0.986	0.837	0.919	0.099
Hw 03	521	277	764.27	0.961	0.851	0.680	0.361
Hw 04	744	535	1229.31	0.963	0.871	0.604	0.434
Hw 05	680	416	1047.48	0.958	0.847	0.648	0.396
Hw 06	1039	901	1876.68	0.969	0.893	0.553	0.480
I 01	3667	388	4007.01	0.989	0.877	0.915	0.097

I 02	2203	257	2426.40	0.988	0.873	0.908	0.106
I 03	483	64	534.33	0.982	0.833	0.902	0.118
I 04	1237	118	1329.65	0.983	0.798	0.930	0.088
I 05	512	42	537.49	0.975	0.648	0.951	0.076
In 01	221	16	228.49	0.976	0.590	0.963	0.066
In 02	209	18	218.62	0.976	0.647	0.951	0.078
In 03	194	14	199.85	0.975	0.553	0.966	0.065
In 04	213	11	217.37	0.983	0.583	0.975	0.046
In 05	188	16	195.65	0.973	0.599	0.956	0.077
Kn 003	1833	74	1891.11	0.993	0.817	0.969	0.039
Kn 004	720	23	733.26	0.991	0.673	0.981	0.030
Kn 005	2477	101	2558.43	0.994	0.829	0.968	0.039
Kn 006	2433	74	2481.41	0.991	0.681	0.980	0.029
Kn 011	2516	63	2557.69	0.993	0.696	0.983	0.024
Lk 01	174	20	184.77	0.967	0.632	0.936	0.103
Lk 02	479	124	579.97	0.967	0.812	0.824	0.212
Lk 03	272	62	317.63	0.960	0.749	0.853	0.192
Lk 04	116	18	125.56	0.958	0.630	0.916	0.135
Lt 01	2211	133	2328.00	0.994	0.898	0.949	0.057
Lt 02	2334	190	2502.00	0.992	0.895	0.932	0.076
Lt 03	2703	103	2783.00	0.993	0.798	0.971	0.037
Lt 04	1910	99	1983.00	0.989	0.757	0.963	0.049
Lt 05	909	33	930.00	0.990	0.704	0.976	0.034
Lt 06	609	19	621.00	0.994	0.760	0.979	0.029
M 01	398	152	526.92	0.963	0.836	0.753	0.287
M 02	277	127	386.01	0.963	0.846	0.715	0.326
M 03	277	128	384.62	0.957	0.823	0.718	0.330
M 04	326	137	444.29	0.966	0.854	0.732	0.306
M 05	514	234	715.18	0.960	0.836	0.717	0.326
Mq 01	289	247	506.98	0.951	0.831	0.568	0.485
Mq 02	150	42	178.59	0.945	0.698	0.834	0.230
Mq 03	301	218	500.37	0.970	0.893	0.600	0.434
Mr 001	1555	75	1612.43	0.991	0.795	0.964	0.046
Mr 018	1788	126	1890.34	0.989	0.827	0.945	0.066
Mr 026	2038	84	2098.93	0.991	0.750	0.970	0.040
Mr 027	1400	92	1467.65	0.986	0.755	0.953	0.062
Mr 288	2079	84	2141.01	0.991	0.764	0.971	0.039
R 01	843	62	886.35	0.983	0.729	0.950	0.069
R 02	1179	110	1269.07	0.987	0.836	0.928	0.086
R 03	719	65	770.20	0.986	0.820	0.932	0.083
R 04	729	49	764.36	0.986	0.766	0.952	0.063

R 05	567	46	599.19	0.982	0.744	0.945	0.075
R 06	432	30	451.75	0.984	0.731	0.954	0.064
Rt 01	223	111	315.91	0.954	0.819	0.703	0.348
Rt 02	214	69	264.75	0.946	0.730	0.805	0.257
Rt 03	207	66	255.86	0.948	0.738	0.805	0.254
Rt 04	181	49	215.58	0.950	0.719	0.835	0.223
Rt 05	197	74	250.69	0.935	0.706	0.782	0.291
Ru 01	422	31	441.04	0.980	0.679	0.955	0.068
Ru 02	1240	138	1356.70	0.987	0.858	0.913	0.101
Ru 03	1792	144	1909.09	0.988	0.825	0.938	0.075
Ru 04	2536	228	2731.76	0.989	0.865	0.928	0.083
Ru 05	6073	701	6722.04	0.993	0.926	0.903	0.104
Sl 01	457	47	493.72	0.985	0.829	0.924	0.093
Sl 02	603	66	651.09	0.978	0.753	0.925	0.100
Sl 03	907	102	990.94	0.985	0.840	0.914	0.102
Sl 04	1102	328	1404.13	0.984	0.918	0.784	0.233
Sl 05	2223	193	2385.35	0.989	0.849	0.932	0.080
Sm 01	267	159	403.17	0.953	0.825	0.660	0.392
Sm 02	222	103	303.92	0.944	0.770	0.727	0.336
Sm 03	140	45	168.39	0.925	0.624	0.825	0.261
Sm 04	153	78	214.17	0.939	0.760	0.710	0.360
Sm 05	124	39	149.49	0.934	0.664	0.823	0.254
T 01	611	89	680.99	0.977	0.802	0.896	0.129
T 02	720	107	807.46	0.980	0.830	0.890	0.131
T 03	645	128	748.50	0.972	0.811	0.860	0.170

The discriminative ability of individual indicators is different. B_1 is always greater than 0.92, visually the differences are not too great but if we consider all computed points, as presented in Figure 5.2, one can see that there is a certain order signalizing a decrease of B_1 with increasing analyticity of language. The Polynesian languages are in the lower part, the most synthetic languages are in the upper part of the figure.

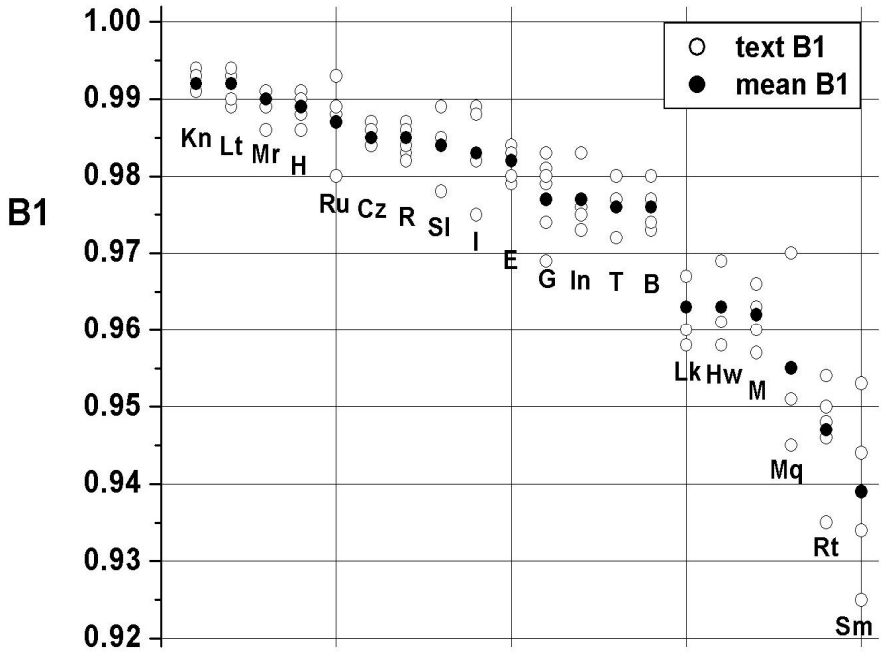


Figure 5.2. The indicator B_1 for 20 languages

The languages ordered according to mean B_1 are presented in Table 5.2. In the last column the value 1000*s* (1000 times the standard deviation) is shown. As can be seen, except for some outliers this value increases in a slight exponential manner. This may be caused either by the small number of texts in individual languages or by some lack of stability of arc length increasing with increasing analyticity. From the linguistic point of view the behaviour of mean B_1 is understandable and will be touched later on, but that of the standard deviation is preliminarily not explainable and must be further scrutinized. Perhaps a variance analysis based on more texts from all languages would help to interpret the behaviour of the standard deviation.

Table 5.2
Means of B_1 for individual languages

	Language	mean B_1	1000*s*
1	Kannada	0.992	1.2
2	Latin	0.992	1.9
3	Marathi	0.990	2.0
4	Hungarian	0.989	1.7
5	Russian	0.987	4.2
6	Czech	0.985	1.2
7	Romanian	0.985	1.8
8	Slovenian	0.984	3.5
9	Italian	0.983	5.0

10	English	0.982	1.6
11	German	0.977	4.9
12	Indonesian	0.977	3.4
13	Tagalog	0.976	3.3
14	Bulgarian	0.976	2.4
15	Lakota	0.963	4.1
16	Hawaiian	0.963	4.0
17	Maori	0.962	3.1
18	Marquesan	0.955	10.7
19	Rarotongan	0.947	6.4
20	Samoan	0.939	9.4

The indicator B_2 is severely normalized and does not display any typological tendency. The ordered values of mean B_2 are presented in Table 5.3 and displayed graphically in Figure 5.3.

Table 5.3
Means of B_2 for individual languages

	Language	mean B_2
1	Hungarian	0.900
2	English	0.878
3	Hawaiian	0.866
4	Maori	0.839
5	Slovenian	0.838
6	Czech	0.835
7	Russian	0.831
8	Tagalog	0.814
9	Marquesan	0.807
10	Italian	0.806
11	Latin	0.802
12	Marathi	0.778
13	Romanian	0.771
14	Rarotongan	0.742
15	Kannada	0.739
16	Samoan	0.729
17	Lakota	0.706
18	German	0.625
19	Indonesian	0.594
20	Bulgarian	0.574

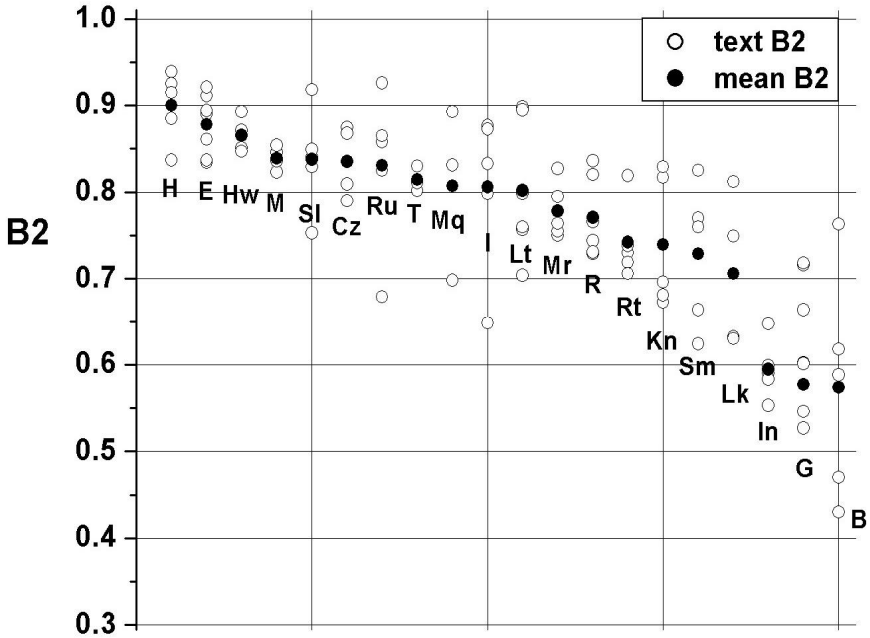


Figure 5.3. The indicator B_2 for 20 languages

The last two indicators, B_3 and B_4 are (almost) complementary because $B_3 + B_4 \approx 1$. Actually,

$$B_3 + B_4 = (V - 1 + f(1) - 1)/L \approx L_{max}/L \approx (L + ph)/L = 1 + p(h/L)$$

where p is a constant of the order of unity as it will be shown in continuation below. The computations of the means of B_3 and B_4 for individual languages are presented in Table 5.4 and in Figures 5.4 and 5.5. As can be seen, strongly analytic languages are in the lower part of the Table, hence these indicators could be used for typological purposes. However, the other languages represent rather a homogeneous mass which could be discriminated using a further variable. This must be left as a task for the future; perhaps the independent Greenberg-Krupa indices could be of help. As to the standard deviation, we have the same case as above but this problem must be solved rather with many texts in fewer languages. Maybe the difference in genres within a language is the cause of the given extent of variability.

Table 5.4
Means of B_3 and B_4 for individual languages

	Language	mean B_3	mean B_4
1	Kannada	0.976	0.032
2	Indonesian	0.962	0.066
3	Latin	0.962	0.047

4	Marathi	0.961	0.051
5	Bulgarian	0.960	0.067
6	German	0.958	0.069
7	Romanian	0.944	0.073
8	Russian	0.927	0.086
9	Italian	0.921	0.097
10	Czech	0.914	0.102
11	Slovenian	0.896	0.122
12	Lakota	0.882	0.161
13	Tagalog	0.882	0.143
14	Hungarian	0.880	0.133
15	English	0.831	0.187
16	Rarotongan	0.786	0.275
17	Samoan	0.749	0.321
18	Maori	0.727	0.315
19	Marquesan	0.667	0.383
20	Hawaiian	0.621	0.418

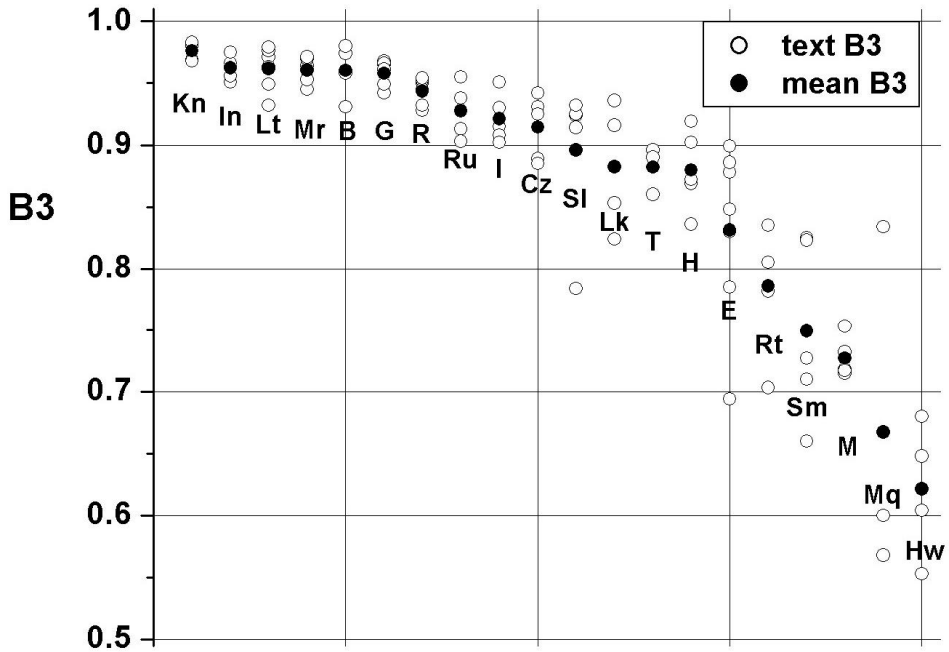


Figure 5.4. The indicator B_3 for 20 languages

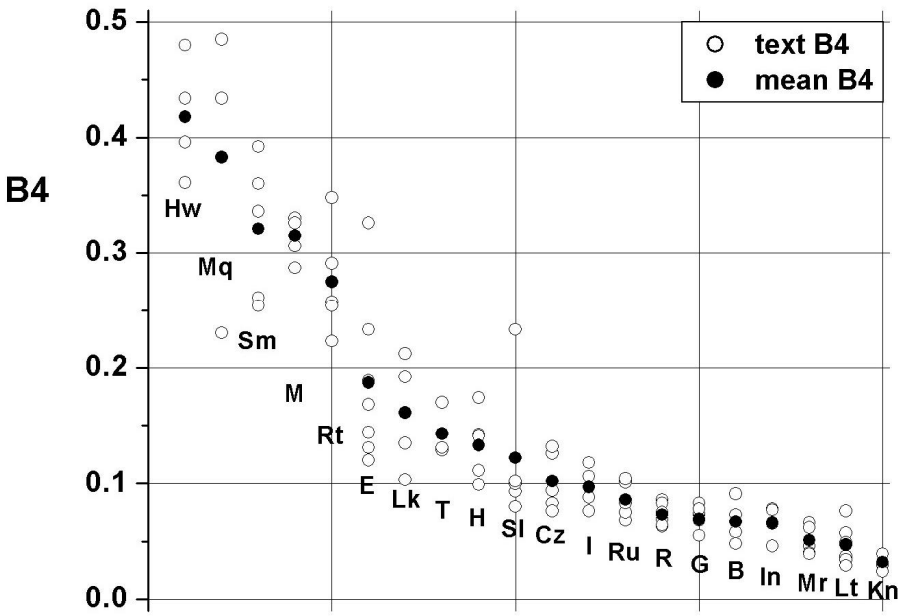


Figure 5.5. The indicator B_4 for 20 languages

5.2. Arc development

Since with increasing text both $f(1)$ and V increase, the arc length (both the empirical L and the theoretical L_{min} as well as L_{max}) increases, too. This change influences in turn some indicators whose development may be relatively regular. Let us consider here only B_3, which takes into consideration only the change of vocabulary (V) and omits the other, pre-h, domain of the arc. Since L which takes into account both parts, is in the denominator, B_3 must decrease with increase of text length. As shown above, the indicator is dependent on the morpho-syntactic character of language, hence for illustration one must take texts from one language only. We have evaluated 52 German texts as shown in Table 5.5 (from Popescu, Mačutek, Altmann 2008). There is a clear trend which can be captured by the power function

$$B_3 = 1.1563N^{-0.0317}$$

yielding a determination coefficient $R^2 = 0.73$ (and a highly significant F-test) which is sufficient because we used very heterogeneous texts. The dependence is presented in Figure 5.6.

Table 5.5
Dependence of B_3 on text length N

ID	N	V	$f(1)$	L	B_3
G 01	1095	530	83	598.77	0.883
G 02	845	361	48	396.78	0.907
G 03	500	281	33	304.25	0.920
G 04	545	269	32	289.67	0.925
G 05	559	332	30	351.41	0.942
G 06	545	326	30	346.82	0.937
G 07	263	169	17	178.72	0.940
G 08	965	509	39	534.55	0.950
G 09	653	379	30	398.43	0.949
G 10	480	301	18	309.84	0.968
G 11	468	297	18	306.80	0.965
G 12	251	169	14	175.44	0.958
G 13	460	253	19	262.25	0.961
G 14	184	129	10	132.54	0.966
G 15	593	378	16	385.09	0.979
G 16	518	292	16	299.24	0.972
G 17	225	124	11	127.96	0.961
G 18	356	227	15	234.23	0.965
G 19	986	561	37	585.28	0.957
G 20	683	411	35	436.46	0.939
G 21	715	421	28	438.45	0.958
G 22	929	502	33	523.39	0.957
G 23	1328	718	53	756.35	0.948
G 24	717	449	40	477.48	0.938
G 25	2025	1024	85	1088.95	0.939
G 26	2063	1029	97	1106.22	0.929
G 27	4047	963	147	1078.03	0.892
G 28	2326	681	100	758.85	0.896
G 29	1630	512	81	575.28	0.888
G 30	1096	374	53	412.00	0.905
G 31	4412	1052	157	1177.79	0.892
G 32	1649	570	69	621.21	0.916
G 33	4515	1051	157	1175.11	0.894
G 34	2909	1036	132	1146.45	0.903
G 35	3253	841	143	956.11	0.879
G 36	5490	1343	270	1579.28	0.850

G 37	6869	1463	315	1736.34	0.842
G 38	4043	1148	178	1296.99	0.884
G 39	3834	1483	126	1583.03	0.936
G 40	2617	1035	94	1196.08	0.864
G 41	3709	1354	147	1473.74	0.918
G 42	3012	1264	127	1368.00	0.923
G 43	7110	2469	276	2706.89	0.912
G 44	5486	1824	257	2048.33	0.890
G 45	9788	2614	454	3023.68	0.864
G 46	12656	3073	591	3612.01	0.850
G 47	6901	1939	329	2227.08	0.870
G 48	9493	2385	485	2823.88	0.844
G 49	12879	2951	656	3553.43	0.830
G 50	8426	2276	403	2637.29	0.863
G 51	8704	2413	406	2774.59	0.869
G 52	12335	3042	596	3589.93	0.847

The regularity of arc development can be seen in some other relationships, too. For example the development of the arc in dependence on f_1 for our German data yields a function $L = 32.8893 f_1^{0.7302}$ with $R^2 = 0.96$, as can easily be computed from Table 5.5. But since in typologically different languages f_1 develops differently, the parameters of the function will be different. Consider the arc length development in individual texts in four languages as shown in Figure 5.7 (from Popescu, Mačutek, Altmann 2008). Evidently, the more synthetic a language, the steeper the slope (or the greater the exponent) of the power function.

This exponent could serve not only as a simple typological indicator but also as a possible indicator of the transition of a language from one “type” to another. The historical study of some French texts and their comparison with Latin ones would be very illuminating. Some German philologists say that there is an analytic tendency in the development of German; other ones deny it. Perhaps this view could help to solve some problems associated with Zipf´s “grand cycle of language evolution”.

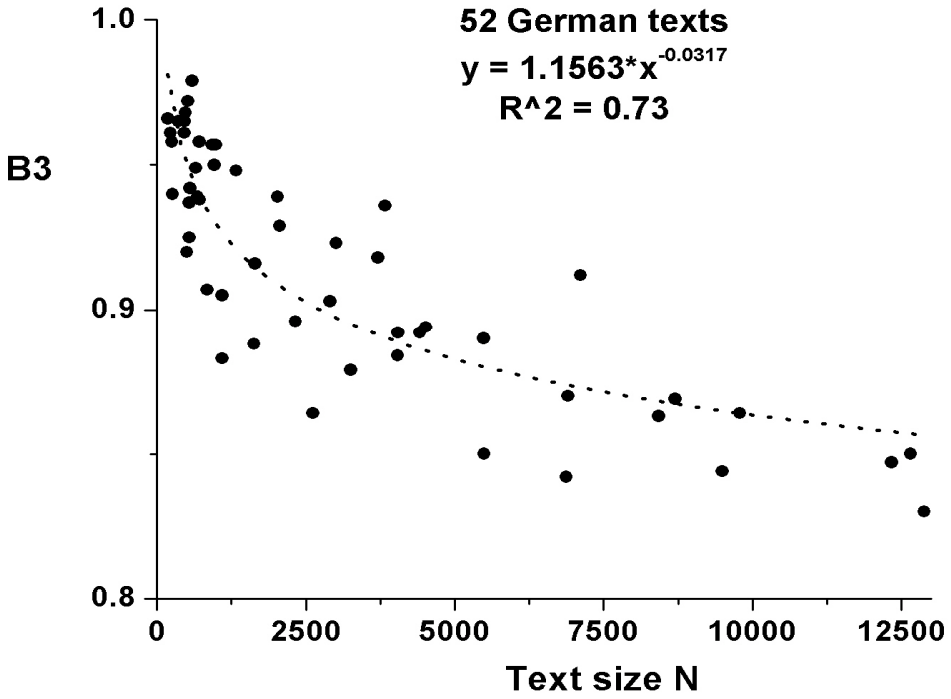


Figure 5.6. Dependence of B_3 on N

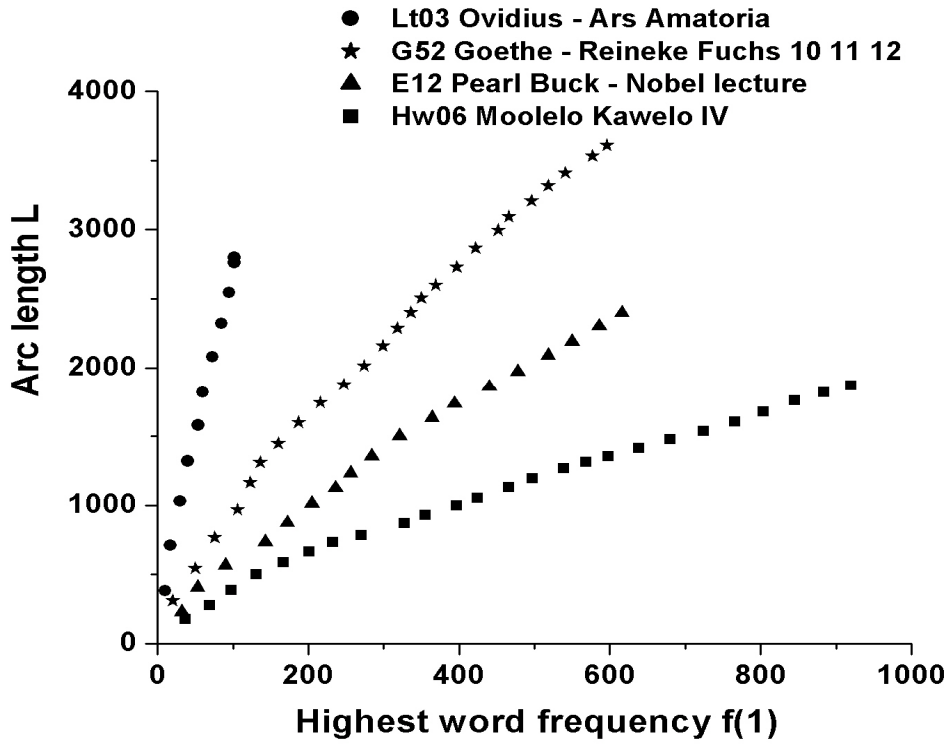


Figure 5.7. Arc length development in cumulative steps of 500 words in terms of f_1 for individual Hawaiian, English, German and Latin texts

5.3. Arc length as a function of text indicators

In all above dependences some empirical constants appeared whose linguistic interpretation was not possible. As a matter of fact, capturing a dependence in this way is a play with the ceteris paribus conditions. The constants indicate that there is still something else having a constant influence on the dependent variable through the independent variable. In this sense, for instance, Table 5.18 with word form indicators of 100 texts, selected from 20 languages, demonstrates empirically the following relationship

(5.24) $L = L_{\max} - p(h - 1)$

where p is a constant of the order of unity, telling us that the difference between the arc length L and its maximum possible value $L_{\max}$ is always of the order of h. As needed, in the extreme case of $h = 1$ we have $L = L_{\max}$. The relationship (5.24) seems to be a basic textual property involving the h-point and has previously been verified in a slightly simplified form, as

(5.25) $L = V + f(1) - ch$,

where c is also a constant of the order of unity (Popescu, Mačutek, Altmann 2008). As a matter of fact, Eq. (5.24) is a relationship between the arc length L, both V and f_1 (through $L_{\max}$) as well as the h-point, all of which developing regularly with increasing N.

In the following applications we shall generalize the relationship (5.24) in order to compute the coefficient

(5.26) $$p = \frac{L_{\max} - L}{h - 1}$$

for any rank-frequency distribution $f = f(r)$, where

(5.27) $L_{max} = (R - 1) + f(1) - f(R)$

is the ideal maximum arc length, $r_{\max} = R$ the maximum rank, $f(1)$ the maximum frequency, and $f(R)$ the minimum frequency. For the sake of comparison we shall generalize also the equation (5.25) in the form

(5.28) $L = R + f(1) - f(R) + 1 - ch$

from which we get the coefficient

(5.29) $$c = \frac{R + f(1) - f(R) + 1 - L}{h}$$

as used in a previous article on the regularity of diversification in language (Popescu, Altmann 2008). Clearly, index c is only an approximation of index p for $h >> 1$, as it results from their very definitions (5.26) and (5.29). This also can easily be seen from the relationship

(5.30) $p = (ch - 2)/(h - 1)$

joining the indicators p and c. In the applications to follow in continuation we shall see that the major difference between the two numbers (coefficients) introduced above is that p is constricted more closely to unity than c, as illustrated in Table 5.6 and in Figure 5.8 summarizing the results of the analysis of 805 rank-frequency distributions of various categories. The means of p and c are denoted by $\overline{p}$ and $\overline{c}$ respectively and the corresponding standard deviations by s_p and s_c.

Table 5.6
Comparison of mean $\overline{p}$ and mean $\overline{c}$
(ranked by increasing mean $\overline{p}$)

Table	**Category** $\boxed{\overline{p}}$	$\overline{p}$	s_p	$\overline{c}$	s_c
5.7	1. Sounds, phonemes and letters	1.013	0.025	1.049	0.025
5.8	2. Word classes	1.024	0.090	1.139	0.080
5.9	3. Rhythmic patterns (Latin, Greek, German)	1.062	0.112	1.141	0.111
5.10	4. Pitches of 58 musical texts	1.086	0.103	1.132	0.098
5.11	5. Colour classes	1.087	0.072	1.185	0.068
5.12	6. Allomorphs of German plural	1.105	0.218	1.353	0.197
5.13	7. Polish paradigmatic classes	1.111	0.053	1.152	0.053
5.14	8. Auxiliaries	1.131	0.130	1.243	0.108
5.15	9. Word frequencies for 24 German authors	1.166	0.099	1.223	0.085
5.16	10. Affixes (meaning diversification)	1.174	0.026	1.386	0.167
5.17	11. English words (meaning diversification)	1.189	0.194	1.456	0.181
5.18	12. Word frequencies for 20 languages	1.223	0.132	1.292	0.116
5.19	13. French *et*	1.234		1.362	
5.20	14. German genitive	1.259		1.335	
	15. French word associations	1.262	0.150	1.422	0.136

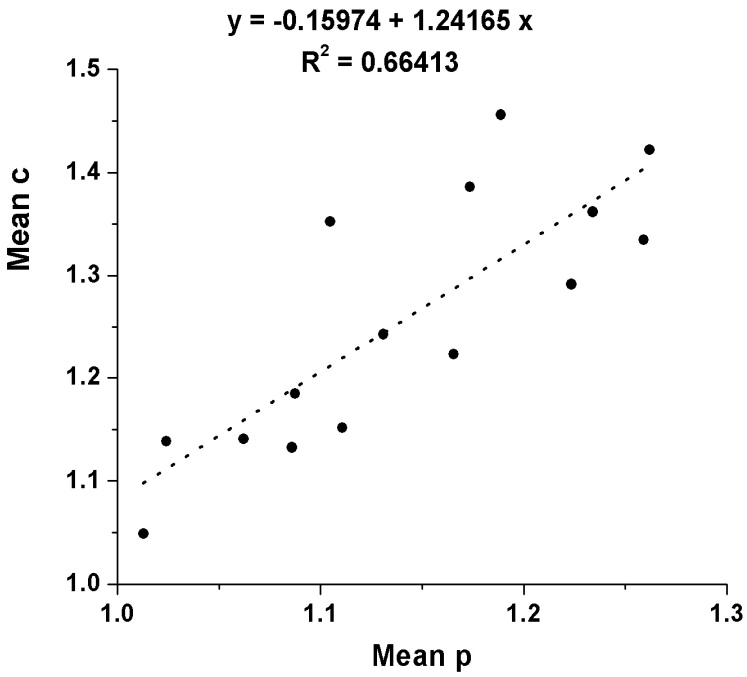


Figure 5.8. Mean $\overline{c}$ versus mean $\overline{p}$ resulting from the analysis of 805 rank-frequency distributions of various categories

For the time being it is not possible to decide about the greater "adequacy" of *p* or *c*. They are evidently correlated and further analyses of linguistic phenomena – whose number is infinite – will surely improve the result in Figure 5.8 which is quasi-linear. The ordering of phenomena according to *p* (Table 5.6) shows that the smaller *p*-values are reserved to phenomena belonging to large classes embracing raw classifications of linguistic entities being rather of formal character. The phenomena with greater *p* are rather specific or individual phenomena. Of course, this is merely the first impression. Many languages and many phenomena must be analyzed in order to find the subtle mechanism, the boundary conditions, the ceteris paribus phenomena etc. controlling the interplay of forces in text.

In the sequel we shall present the data in that order in which they appear in Table 5.6.

5.4. Analysis of language levels

In what follows we present and comment on all data that were at our disposal. It is to be emphasized that all these data are categorical (nominal, classificatory) and our aim is to show that there is a certain order in "adequately" determined categories. The argument holds in both directions: if we define a linguistic class, then its rank-frequency distribution must abide by some regularities; and vice

versa, if in a class we find the given regularity, then all the given elements belong to the class. This is a kind of remedy against fuzzy membership which destroys many grammatical classifications. If an element belongs to class A in degree 70 and to class B in degree 40 – a quite usual situation in linguistics – then the next possible objective criterion for decision is its place in the rank-frequency sequence in both classes: that class is more adequate in which its presence "improves" the above mentioned situation.

The data at our disposal concern very different phenomena in a number of languages, so that one can get only a first sight of the phenomenon. Hence, all results must be taken cum grano salis and further data must be collected in order to corroborate whatever hypothesis thereon. We prefer to speak about language phenomena rather than about language levels because the present investigation only partially corresponds with our rather intuitive conception of language level.

5.4.1. Sounds, phonemes and letters. Let us begin with **sounds, phonemes** and **letters** which do not have their own meaning. In order to avoid the well known problems associated with the identification of these units, we took published data from different languages. All Russian data were taken from Grzybek, Kelih (2003). The English data were obtained from texts available on the Internet (http://www.gutenberg.org/browse/scores/top). Fry (1947) was considered on historical grounds. The number of different counts available on the Internet is enormous.

The data and the results of computation are presented in Table 5.7. As can be seen, the value of *c* is stable both historically – as shown in Russian data from different years – and cross-linguistically, hence, it is no typological feature but, perhaps, characterizes the rank-frequency distribution on a certain level.

Table 5.7
Computing the *c* and *p* coefficients for sounds, phonemes and letters in different languages
$\overline{p} = 1.013$, $s_p = 0.025$, $\overline{c} = 1.049$, $s_c = 0.025$

Source	R	$f(1)$	$f(R)$	h	L	L_{max}	p	c
Ch. Dickens, David Copperfield, letters	26	181444	267	25.98	181177	181202	1.001	1.039
Ch. Dickens, Great Expectations, letters	26	91775	209	25.95	91566	91591	1.002	1.040
Ch. Dickens, A Christmas Carol, letters	26	14914	86	24.19	14829	14853	1.035	1.075
J. Joyce, Ulysses, letters	26	141465	1077	25.90	140388	140413	1.004	1.042
C. Doyle, Sherlock Holmes, letters	26	53034	148	25.91	52886	52911	1.004	1.042

M. Twain, Huckle-berry Finn, letters	26	47144	178	24.92	46966	46991	1.045	1.083
J. Milton, Paradise Lost, letters	26	42728	176	25.67	42552	42577	1.013	1.052
H.G. Wells, The War of the Worlds, letters	26	33398	105	25.67	33293	33318	1.013	1.052
J. Swift, Gulliver's Travels, letters	26	58078	145	25.93	57933	57958	1.003	1.041
E. Bronte, Wuther-ing Heights, letters	26	63773	198	25.91	63575	63600	1.004	1.042
Ch. Bronte, Jane Eyre, letters	26	100613	328	25.96	100285	100310	1.002	1.040
B. Stoker, Dracula, letters	26	79310	351	25.91	78959	78984	1.004	1.042
English sounds (Fry 1947)	44	51830	334	43.84	51496	51539	1.004	1.026
Finnish letters (Pääkkönen 1994)	27	296538	25	25.93	296513	296539	1.043	1.080
Georgian phonemes (Job 1974)	33	2064	9	29.52	2057	2087	1.052	1.084
German sounds (Meier 1964; Best 2004/2005)	46	10275	1	43.79	10275	10319	1.028	1.050
Hawaiian letters (Schulze 1974)	13	5305	80	12.61	5225	5237	1.034	1.110
Sea Dayak letters (Rademacher 1974)	21	4428	15	19.94	4414	4433	1.003	1.053
Slovenian letters (Grzybek, Kelih 2006)	25	32036	497	24.98	31539	31563	1.001	1.041
Slovak letters (Grzy-bek, Kelih 2006)	42	14193	3	39.29	14190	14231	1.071	1.094
Serbian letters (Grzybek, Kelih 2006)	29	885	14	26.38	875	899	0.946	0.986
Russian letters (Ol´chin 1907)	29	2460	50	28.26	2411	2438	0.990	1.026
Russian letters (Proskurin 1933)	33	110020	331	32.38	109689	109721	1.020	1.050
Russian letters (Kalinina 1968)	31	11376	422	29.83	10954	10984	1.041	1.073
Russian letters Grigor´ev 1980a)	32	5678	16	30.97	5663	5693	1.001	1.033
Russian letters (Grigor´ev 1980b)	32	11410	22	31.48	11389	11419	0.984	1.017

Russian letters (Dietze 1982)	32	44172	156	31.97	44016	44047	1.001	1.032

5.4.2. Word classes. Word classes defined in classical Latin manner have been published by different authors. The classification of words in classes can be performed in different ways resulting in dozens of classes, but it is not our aim to compare them. We simply study the forming of the rank-frequency distribution in a possible classification. The data and the results are presented in Table 5.8. The data were taken from the following sources: Latin, German, Chinese (Schweers, Zhu 1991), Polish (Sambor 1989), German (Best 1994), Portuguese, Brazilian Portuguese (Ziegler 2001).

Table 5.8
Computing the *c* and *p* coefficients for word classes
$\overline{p}$ = 1.024, s_p = 0.090, $\overline{c}$ = 1.139, s_c = 0.080

Language	*R*	*f*(*1*)	*f*(*R*)	*h*	*L*	L_{max}	*p*	*c*
Latin	9	347	9	8.74	339	346	0.956	1.076
German	8	192	70	7.75	123	129	0.938	1.075
Chinese	8	247	27	7.95	220	227	0.968	1.098
Polish	10	144188	650	8.73	143538	143547	1.164	1.260
German (Best)	10	2032	761	10.00	1271	1280	0.997	1.097
Portuguese	9	2586	352	8.91	2234	2242	1.006	1.118
Brazilian Portuguese	9	2930	394	8.00	2536	2544	1.139	1.246

This way of determining word classes, though not the only possible one, yields a very compact result. In Portuguese numerals were considered a separate word class. In German (Best 1994) the unique interjection has been omitted. As can be seen, the text size (*N*) does not play any role. There is no difference between languages, hence the quantities *c* and *p* cannot be used for typological purposes. It is a matter of the given phenomenon. A "last moment" parts-of-speech result of 60 End-of-Year Addresses of Italian presidents, beginning with Einaudi in 1949 and ending with Napolitano in 2008, gives $\overline{p}$ = 1.062 with s_p = 0.070 (Tuzzi, Popescu, Altmann 2009).

5.4.3. Rhythmic units. Rhythmic units are based mostly on suprasegmental features and have nothing common either with grammar or with semantics. We use here line patterns in Latin, Greek and German hexameter and distych. The data were taken from Drobisch (1866, 1872, 1875, 1868a,b) and presented for other purposes in Best (2008). Since the last two feet in the verse are identical, it

is sufficient to consider the combinations of dactyls (D) and spondees (S) in the first four positions. One obtains 16 patterns like SSSS, SSSD, SSDS, SDSS,… The authors may differ in the use of individual patterns whose identity is here irrelevant, we consider only the rank order of patterns. All data are samples of uninterrupted sequences from the works of the given authors. We obtained from Drobisch the data whose evaluation is presented in Table 5.9.

Table 5.9
Computing the *c* and *p* coefficients for hexameter patterns in Latin, Greek and German (Drobisch 1866, 1868a,b, 1872, 1985)
$\overline{p}$ = 1.062, s_p = 0.112, $\overline{c}$ = 1.141, s_c = 0.111

Text	R	$f(1)$	$f(R)$	h	L	L_{max}	p	c
Goethe, “Reinecke Fuchs”	16	204	5	12.67	201	214	1.114	1.184
Goethe, “Hermann und Dorothea”	16	200	11	14.09	192	204	0.917	0.994
Goethe, “Elegien”	16	96	4	9.78	96	107	1.253	1.329
Leibniz, “Epicedium”	16	59	2	10.33	62	72	1.072	1.162
Klopstock,: “Messias”	16	129	3	13.78	129	141	0.939	1.016
Voss, “Luise”	14	188	9	13.46	180	192	0.963	1.040
Voss, “Odyssey”	16	125	1	14.15	126	139	0.989	1.060
Vergil, “Georgica”	16	84	9	11.00	80	90	1.000	1.091
Vergil, other sample	16	78	4	13.33	76	89	1.054	1.125
Vergil, “Aeneis“	16	423	58	13.87	367	380	1.010	1.081
Vergil, “Bucolica”	16	107	21	11.70	89	101	1.121	1.197
Horace, another sample	16	62	11	12.00	56	66	0.909	1.000
Horace, “Satires”	16	285	46	14.25	240	254	1.057	1.123
Horace, “Epistulae”	16	237	35	15.21	203	217	0.985	1.052
Lucrece, “De rerum natura”	16	88	7	11.00	84	96	1.200	1.273
Manilius, “Astronomica”	16	93	9	11.75	88	99	1.023	1.106
Persius, “Satires”	16	118	5	12.50	116	128	1.043	1.120
Juvenal, “Satires”	16	85	12	11.00	76	88	1.200	1.273
Lucanus, “Pharsalia”	16	98	8	11.20	93	105	1.176	1.250
Quintus Ennius, “Fragments”	16	64	10	11.00	61	69	0.800	0.909
Catull, 2 poems	16	124	1	8.88	128	138	1.269	1.351
Ovid, “Metamorphoses”	16	78	5	10.78	77	88	1.125	1.206
Silius Italicus, “Punica”	16	75	7	12.86	72	83	0.927	1.011
Valerius Flaccus, “Argonautica”	16	131	3	12.70	131	143	1.026	1.102
Statius, “Thebais”	16	83	8	10.88	78	90	1.215	1.287
Claudian,: “Raptus Proserpinae”	16	102	2	11.29	103	115	1.166	1.240
Homer, “Iliad”	16	350	8	14.00	343	357	1.077	1.143
Homer, “Odyssey”	16	410	5	14.67	406	420	1.024	1.091

Theokrit, “1st Idyll“	13	31	1	7.00	35	42	1.167	1.286
Theognis, “Elegic poems“	16	117	2	12.53	118	130	1.041	1.117

5.4.4. Pitches of 58 musical texts of 12 European composers. In order to compare some other texts and entities we computed the necessary quantities for 58 musical texts in which we considered the frequencies of pitches (cf. Martinaková, Popescu, Mačutek, Altmann 2008). The results are presented in Table 5.10.

Table 5.10
Computing the *c* and *p* coefficients for 58 musical texts of 12 European composers
$\overline{p}$ = 1.086, s_p = 0.103, $\overline{c}$ = 1.132, s_c = 0.098

ID	N	R	$f(1)$	$f(R)$	h	L	L_{max}	p	c
Bach01	1318	44	106	1	21	124	148	1.200	1.238
Bach02	1877	45	147	1	25	163	190	1.125	1.160
Bach03	2266	45	155	1	24	169	198	1.261	1.292
Bach04	2085	47	140	1	25	158	185	1.125	1.160
Bach05	1553	44	113	1	23	129	155	1.182	1.217
Beethoven01	7332	59	537	1	42	550	594	1.073	1.095
Beethoven02	9340	62	626	1	45	644	686	0.955	0.978
Beethoven03	11915	63	625	1	49	636	686	1.042	1.061
Beethoven04	12248	63	868	1	50	879	929	1.020	1.040
Beethoven05	7229	63	473	1	42	492	534	1.024	1.048
Gesualdo01	688	35	65	1	15	83	98	1.071	1.133
Gesualdo02	591	34	51	1	15	68	83	1.071	1.133
Gesualdo03	581	37	52	1	14	72	87	1.154	1.214
Gesualdo04	761	36	67	1	17	82	101	1.188	1.235
Gesualdo05	671	33	61	1	16	76	92	1.067	1.125
Ligeti01	3017	74	107	1	38	146	179	0.892	0.921
Ligeti02	3142	85	125	1	36	174	208	0.971	1.000
Ligeti03	3015	74	101	1	38	140	173	0.892	0.921
Liszt01	1495	65	128	1	22	166	191	1.190	1.227
Liszt02	4278	75	400	1	35	433	473	1.176	1.200
Liszt03	3003	78	113	1	37	152	189	1.028	1.054
Liszt04	4420	71	273	1	38	298	342	1.189	1.211
Liszt05	2899	65	287	1	30	317	350	1.138	1.167
Monteverdi01	3002	37	399	1	21	408	434	1.300	1.333

Monteverdi02	1927	30	242	1	20	248	270	1.158	1.200
Monteverdi03	2719	35	240	1	24	249	273	1.043	1.083
Monteverdi04	3138	32	341	1	23	347	371	1.091	1.130
Monteverdi05	2161	32	260	1	20	267	290	1.211	1.250
Mozart01	10585	58	692	1	40	707	748	1.051	1.075
Mozart02	7577	56	482	1	35	495	536	1.206	1.229
Mozart03	8117	59	499	1	41	510	556	1.150	1.171
Mozart04	7496	57	474	1	35	491	529	1.118	1.143
Mozart05	9470	55	1002	1	36	1012	1055	1.229	1.250
Palestrina01	1856	27	209	1	19	215	234	1.056	1.105
Palestrina02	898	24	101	1	15	108	123	1.071	1.133
Palestrina03	1348	26	157	1	17	164	181	1.063	1.118
Palestrina04	2120	27	243	1	19	248	268	1.111	1.158
Palestrina05	595	23	70	1	14	76	91	1.154	1.214
Schoenberg01	15477	65	913	1	52	924	976	1.020	1.038
Schoenberg02	1197	67	68	1	22	112	133	1.000	1.045
Schoenberg03	1146	63	64	1	22	105	125	0.952	1.000
Schoenberg04	1108	70	51	1	23	97	119	1.000	1.043
Schoenberg05	661	56	50	1	17	87	104	1.063	1.118
Shostakovich01	440	30	73	1	12	87	101	1.273	1.333
Shostakovich02	172	14	26	1	9	30	38	1.000	1.111
Shostakovich03	323	32	33	1	11	51	63	1.200	1.273
Shostakovich04	247	34	23	1	10	45	55	1.111	1.200
Shostakovich05	330	33	57	1	10	78	88	1.111	1.200
Skrjabin01	355	48	19	1	12	55	65	0.909	1.000
Skrjabin02	222	31	20	1	10	42	49	0.778	0.900
Skrjabin03	651	51	33	1	16	67	82	1.000	1.063
Skrjabin04	155	32	12	1	7	36	42	1.000	1.143
Skrjabin05	195	38	23	1	8	52	59	1.000	1.125
Stravinsky01	2490	51	194	1	28	214	243	1.074	1.107
Stravinsky02	5139	64	407	1	33	430	469	1.219	1.242
Stravinsky03	2794	73	182	1	30	220	253	1.138	1.167
Stravinsky04	2805	65	396	1	31	428	459	1.033	1.065
Stravinsky05	3267	71	215	1	37	246	284	1.056	1.081

Comparing all mean $\overline{p}$ we see that German does not differ from the mean $\overline{p}$ of many languages; in music there is a slightly smaller value but in spite of changing musical styles no historical trend is observable, a fact which is a strong corroboration of the constancy of p.

5.4.5. Colour names. Colour names and their frequencies in several languages were studied by A. Pawlowski (1999). Colour names are a closed and relatively small semantic class of adjectives. It has been observed that the rank-frequencies follow a usual law. Pawlowski took into account only colours, hence the data are not complete. Some colours were not present in the frequency dictionary and their number was given as 0. Since this is not the usual way of counting frequencies, the results must be considered cum grano salis. Nevertheless, one can at least have a tentative look at the data. The computation of *c* and *p* is presented in Table 5.11.

Table 5.11
Computing the *c* and *p* coefficients for colour names in 10 languages (Pawlowski 1999)
$\overline{p}$ = 1.087, s_p = 0.072, $\overline{c}$ = 1.185, s_c = 0.068

Language	R	$f(1)$	$f(R)$	h	L	L_{max}	p	c
Czech	12	604	3	10.31	601.98	612	1.076	1.166
English	12	365	7	9.73	358.81	369	1.167	1.253
French (Juilland)	12	136	0	8.00	139.40	147	1.086	1.200
French (Engwall)	12	298	0	10.00	299.50	309	1.056	1.150
Italian	12	155	0	10.17	156.46	166	1.040	1.135
Polish	12	93	0	9.00	94.93	104	1.134	1.230
Romanian	12	165	0	7.63	169.40	176	0.995	1.127
Russian	12	473	16	10.55	457.54	468	1.095	1.181
Slovak	12	473	7	11.15	467.22	477	0.964	1.057
Spanish	12	141	0	7.94	143.67	152	1.200	1.301
Ukrainian	12	310	0	10.14	310.51	321	1.148	1.232

5.4.6. Allomorphs of the German plural. In two cases the allomorphs of the German plural have been studied. Meuser, Schütte and Stremme (2008) analyzed 21 short stories by Wolfdietrich Schnurre, and Brüers and Heeren (2004) the individual letters of Heinrich von Kleist. The ranks differ in all texts, sometimes not all allomorphs are used in a unique text, but the distributions display the same tendencies. The data and the results are presented in Table 5.12.

Table 5.12
Computing the *c* and *p* coefficients for the allomorphs of the German plural
(Texts I: Meuser, Schütte, Stremme (2008); Texts II: Brüers, Heeren (2004))
$\overline{p}$ = 1.105, s_p = 0.218, $\overline{c}$ = 1.353, s_c = 0.197

Texts I	*R*	*f*(1)	*f*(*R*)	*h*	*L*	L_{max}	*p*	*c*
1	8	9	1	5.00	11.54	15	0.865	1.092
2	6	28	3	5.17	25.89	30	0.986	1.182
3	8	17	2	4.66	18.53	22	0.948	1.174
4	7	20	1	4.00	21.37	25	1.210	1.408
5	7	11	1	5.00	11.98	16	1.005	1.204
6	8	20	2	3.33	21.92	25	1.322	1.526
7	8	21	2	4.75	21.27	26	1.261	1.417
8	8	59	7	6.00	53.77	59	1.046	1.205
9	8	12	1	4.80	14.35	18	0.961	1.177
10	8	51	1	5.00	51.61	57	1.348	1.478
11	8	20	1	5.50	21.40	26	1.022	1.200
12	7	20	4	3.50	19.28	22	1.088	1.349
13	7	18	1	5.50	19.24	23	0.836	1.047
14	9	125	4	6.67	122.72	129	1.108	1.241
15	9	10	1	3.50	14.33	17	1.068	1.334
16	9	39	1	6.62	40.08	46	1.053	1.196
17	6	18	1	4.00	18.06	22	1.313	1.485
18	6	18	4	4.33	15.68	19	0.997	1.229
19	8	33	3	5.50	32.07	37	1.096	1.260
20	9	63	2	7.00	63.60	69	0.900	1.057
21	6	11	1	3.00	13.30	15	0.850	1.233
Texts II	*R*	*f*(1)	*f*(*R*)	*h*	*L*	L_{max}	*p*	*c*
1	6	17	1	4.33	17.35	21	1.096	1.305
2	7	8	1	3.50	9.71	13	1.316	1.511
3	7	10	1	2.50	12.90	15	1.400	1.640
4	3	6	1	2.33	5.40	7	1.203	1.545
5	6	12	1	4.00	13.71	16	0.763	1.073
6	4	6	1	2.50	5.99	8	1.340	1.604
7	4	5	2	2.33	4.65	6	1.015	1.438
8	4	9	1	2.50	8.91	11	1.393	1.636
9	6	11	2	3.50	11.01	14	1.196	1.426
10	5	6	1	3.33	6.89	9	0.906	1.234
11	6	5	2	1.75	7.16	8	1.120	1.623
12	7	16	1	3.50	17.41	21	1.436	1.597
13	5	8	1	3.00	8.23	11	1.385	1.590

14	5	4	1	3.00	5.24	7	0.880	1.253
15	4	7	1	2.75	6.81	9	1.251	1.524
16	5	6	2	2.33	6.47	8	1.150	1.515
17	4	7	3	2.67	5.06	7	1.162	1.476
18	8	18	2	4.00	19.23	23	1.257	1.443
19	5	3	1	3.00	5.23	6	0.385	0.923
20	5	7	1	2.00	8.51	10	1.490	1.745
21	6	23	4	4.33	20.75	24	0.976	1.212

5.4.7. Paradigmatic classes. J. Sambor (1989) studied the distribution of **inflection classes of nouns and verbs** in the Polish frequency dictionary. She took two aspects into account: the number of nouns and verbs belonging to a given class and the frequency of the entire class in the dictionary. In this way she obtained four diversification cases:

Number of nouns belonging to a special inflection class
Number of verbs belonging to a special inflection class
Frequencies of individual noun classes
Frequencies of individual verb classes.

The results are given in Table 5.13.

Table 5.13
Computing the *c* and *p* coefficients for the diversification of Polish inflection classes (Sambor 1989)
$\overline{p}$ = 1.111, s_p = 0.053, $\overline{c}$ = 1.152, s_c = 0.053

Classes	***R***	***f*(1)**	***f*(*R*)**	***h***	***L***	L_{max}	***p***	***c***
Nouns, number	28	2624	1	20.50	2627	2650	1.189	1.229
Verbs, number	27	1728	1	18.92	1733	1753	1.095	1.143
Nouns, frequency	28	34288	4	24.79	34285	34311	1.077	1.115
Verbs, frequency	27	19292	1	23.79	19292	19317	1.081	1.120

5.4.8. Auxiliaries. Prepositions, postpositions and conjunctions. Auxiliaries, prepositions, postpositions and conjunctions are auxiliaries underlying strong diversification because they occur frequently and in many contexts. Here specimens from 5 languages are presented in Table 5.14.

Table 5.14
Computing the *c* and *p* coefficients for auxiliaries
$\overline{p}$ = 1.131, s_p = 0.130, $\overline{c}$ = 1.243, s_c = 0.108

Auxiliaries	*R*	*f*(1)	*f*(*R*)	*h*	*L*	L_{max}	*p*	*c*
Japanese: postposition *ni* (Roos 1991)	12	40	1	6.83	42.79	50	1.237	1.348
German: particle/preposition *von* (Best 1991)	53	54	1	11.00	93.08	105	1.192	1.265
German: preposition *auf* (Th.Mann) Fuchs (1991)	27	24	1	6.00	44.60	49	0.880	1.067
German: preposition *auf* (C. Wolf) Fuchs (1991)	54	312	1	14.67	347.03	364	1.241	1.293
English: preposition *in* Hennern (1991)	43	51	1	7.50	84.51	92	1.152	1.265
Polish: preposition *w* Hammerl, Sambor (1991)	12	199	1	9.67	200.42	209	0.990	1.094
Russian: conjunction *no* (Kuße 1991)	16	19	1	7.50	25.62	33	1.135	1.251
Russian: conjunction *a* Kuße (1991).	18	22	1	5.50	32.51	38	1.220	1.362

5.4.9. Word forms of 166 texts of 24 German authors. In order to initiate the continuation of this research we analyzed 166 German texts adhering to the principle that the text parts should be homogeneous, i.e., some texts were divided in subsequent parts, and we made a historical survey from the last two centuries. The analysis is presented in Table 5.15 and the identification of texts is given in the Appendix. Considering the *p*-values in time perspective no trend of any kind can be observed. Hence, in the first step, we may conclude that for texts in classical sense *p* is a text constant not changing in time but reacting to some unknown boundary conditions.

Table 5.15
Computing the *p* and *c* coefficients for word forms of 166 texts of 24 German writers
$\overline{p}$ = 1.166, s_p = 0.099, $\overline{c}$ = 1.223, s_c = 0.085
(in this table the vocabulary *V* means the maximum rank *R*)

ID	*N*	*V*	*f*(1)	*f*(*V*)	*h*	*L*	L_{max}	*p*	*c*
1802noval01	2894	1129	139	1	21	1243	1266	1.139	1.180
1802noval02	3719	1487	208	1	22	1669	1693	1.153	1.192
1802noval03	5321	1819	233	1	25	2018	2050	1.324	1.351
1802noval04	2777	1282	130	1	18	1389	1410	1.259	1.300
1802noval05	8866	2769	473	1	35	3198	3240	1.239	1.261
1802noval06	4030	1467	178	1	23	1617	1643	1.171	1.207
1802noval07	1744	792	77	1	16	851	867	1.070	1.128
1802noval08	2111	816	75	1	17	869	889	1.227	1.272
1802noval09	8945	2681	442	1	32	3082	3121	1.259	1.283
1802noval10	5367	1939	238	1	26	2144	2175	1.226	1.255
1807goeth01	7554	2222	318	1	33	2502	2538	1.124	1.151
1809paul01	854	487	37	1	10	512	522	1.111	1.200
1809paul02	383	255	14	1	6	260	267	1.400	1.500
1809paul03	520	311	26	1	8	326	335	1.286	1.375
1809paul04	580	354	21	1	8	365	373	1.143	1.250
1809paul05	1331	677	44	1	12	705	719	1.273	1.333
1809paul06	526	305	16	1	8	313	319	0.857	1.000
1809paul07	508	316	15	1	7	323	329	1.000	1.143
1809paul08	402	248	22	1	6	262	268	1.200	1.333
1809paul09	1068	547	37	1	10	570	582	1.333	1.400
1809paul10	1558	778	53	1	13	814	829	1.250	1.308
1809paul11	2232	1027	84	1	15	1092	1109	1.214	1.267
1809paul12	620	365	25	1	8	380	388	1.143	1.250
1809paul13	1392	652	40	1	13	676	690	1.167	1.231
1809paul14	1400	714	49	1	14	746	761	1.154	1.214
1809paul15	1648	793	65	1	15	840	856	1.143	1.200
1809paul16	320	223	12	1	5	227	233	1.500	1.600
1809paul17	1844	897	73	1	15	952	968	1.143	1.200
1809paul18	870	489	42	1	11	520	529	0.900	1.000
1809paul19	1236	676	38	1	13	699	712	1.083	1.154
1809paul20	2059	1011	78	1	16	1068	1087	1.267	1.313
1809paul21	3955	1513	172	1	24	1659	1683	1.043	1.083
1809paul22	478	302	15	1	7	309	315	1.000	1.143
1809paul23	656	386	26	1	9	401	410	1.125	1.222
1809paul24	1465	730	80	1	13	795	808	1.083	1.154

1809paul25	588	361	18	1	8	370	377	1.000	1.125
1809paul26	1896	887	61	1	15	930	946	1.143	1.200
1809paul27	749	410	26	1	9	426	434	1.000	1.111
1809paul28	241	172	8	1	5	174	178	1.000	1.200
1809paul29	1825	872	68	1	14	921	938	1.308	1.357
1809paul30	388	238	17	1	6	248	253	1.000	1.167
1809paul31	1630	753	72	1	14	810	823	1.000	1.071
1809paul32	163	119	6	1	4	120	123	1.000	1.250
1809paul33	596	355	23	1	8	369	376	1.000	1.125
1809paul35	1947	897	82	1	17	960	977	1.063	1.118
1809paul36	425	253	15	1	7	259	266	1.167	1.286
1809paul37	368	239	12	1	6	243	249	1.200	1.333
1809paul38	1218	636	40	1	12	660	674	1.273	1.333
1809paul39	388	248	13	1	7	253	259	1.000	1.143
1809paul40	1370	655	53	1	14	694	706	0.923	1.000
1809paul41	1032	546	43	1	11	575	587	1.200	1.273
1809paul42	1546	731	50	1	13	764	779	1.250	1.308
1809paul43	4148	1591	152	1	26	1714	1741	1.080	1.115
1809paul44	1881	896	66	1	15	943	960	1.214	1.267
1809paul45	2723	1102	155	1	18	1236	1255	1.118	1.167
1809paul46	3095	1276	99	1	21	1351	1373	1.100	1.143
1809paul47	516	319	19	1	8	330	336	0.857	1.000
1809paul48	1200	604	50	1	13	638	652	1.167	1.231
1809paul49	562	336	19	1	8	346	353	1.000	1.125
1809paul50	430	255	23	1	7	269	276	1.167	1.286
1809paul51	3222	1323	116	1	20	1413	1437	1.263	1.300
1809paul52	1731	815	71	1	15	870	884	1.000	1.067
1809paul53	1839	864	75	1	14	922	937	1.154	1.214
1809paul54	6644	2417	245	1	30	2625	2660	1.207	1.233
1809paul55	7854	2680	321	1	33	2961	2999	1.188	1.212
1809paul56	963	482	47	1	10	516	527	1.222	1.300
1813chami01	2210	884	82	1	18	944	964	1.176	1.222
1813chami02	1847	808	84	1	16	872	890	1.200	1.250
1813chami03	1428	630	70	1	14	684	698	1.077	1.143
1813chami04	3205	1209	123	1	20	1305	1330	1.316	1.350
1813chami05	2108	853	79	1	18	911	930	1.118	1.167
1813chami06	1948	801	75	1	17	853	874	1.313	1.353
1813chami07	1362	670	44	1	13	698	712	1.167	1.231
1813chami08	1870	788	80	1	16	848	866	1.200	1.250
1813chami09	1320	593	96	1	14	673	687	1.077	1.143
1813chami10	1012	536	52	1	11	575	586	1.100	1.182

1813chami11	1386	656	66	1	14	705	720	1.154	1.214
1817hoffm01	2974	1176	95	1	22	1247	1269	1.026	1.070
1817hoffm02	1076	534	29	1	11	549	561	1.228	1.298
1817hoffm03	8163	2511	290	1	34	2759	2799	1.206	1.229
1818arnim01	7846	2221	271	1	33	2448	2490	1.308	1.329
1824heine01	19522	5769	939	1	46	6648	6706	1.285	1.300
1826eiche01	3080	1079	177	1	21	1228	1254	1.300	1.333
1826eiche02	4100	1287	210	1	25	1466	1495	1.208	1.240
1826eiche03	4342	1334	182	1	28	1482	1514	1.185	1.214
1826eiche04	1781	739	79	1	16	799	816	1.133	1.188
1826eiche05	1680	699	70	1	16	750	767	1.133	1.188
1826eiche06	3223	1059	130	1	22	1163	1187	1.143	1.182
1826eiche07	2594	932	121	1	20	1031	1051	1.053	1.100
1826eiche08	3987	1320	159	1	25	1447	1477	1.250	1.280
1826eiche09	3285	1185	155	1	22	1315	1338	1.095	1.136
1826eiche10	3052	1073	131	1	22	1178	1202	1.143	1.182
1830immer01	28943	6397	918	1	63	7234	7313	1.281	1.292
1842drost01	16172	4064	525	1	49	4528	4587	1.236	1.252
1843seals01	1352	600	45	1	13	629	643	1.167	1.231
1843seals02	4663	1825	142	1	27	1936	1965	1.115	1.148
1843seals03	3238	1197	114	1	21	1284	1309	1.250	1.286
1843seals04	3954	1399	161	1	24	1530	1558	1.217	1.250
1843seals05	3187	1079	96	1	22	1149	1173	1.143	1.182
1843seals06	2586	1010	67	1	20	1053	1075	1.158	1.200
1843seals07	2939	1035	75	1	20	1086	1108	1.158	1.200
1843seals08	4865	1333	138	1	27	1435	1469	1.308	1.333
1843seals09	7259	2295	263	1	31	2519	2556	1.233	1.258
1843seals10	4838	1620	138	1	26	1726	1756	1.200	1.231
1843seals11	3785	1265	98	1	26	1333	1361	1.120	1.154
1843seals12	3019	1191	95	1	20	1262	1284	1.158	1.200
1843seals13	2370	1071	89	1	17	1139	1158	1.188	1.235
1843seals14	2744	1198	82	1	19	1257	1278	1.167	1.211
1843seals15	4786	1545	164	1	27	1676	1707	1.192	1.222
1843seals16	4497	1602	137	1	26	1707	1737	1.200	1.231
1843seals17	6705	2273	192	1	30	2429	2463	1.172	1.200
1843seals18	4162	1252	285	1	24	1508	1535	1.174	1.208
1843seals19	5626	1653	171	1	29	1789	1822	1.179	1.207
1843seals20	8423	2735	273	1	35	2966	3006	1.176	1.200
1843seals21	6041	2040	220	1	29	2224	2258	1.214	1.241
1843seals22	5748	1655	157	1	29	1776	1810	1.214	1.241
1843seals23	1752	799	80	1	14	861	877	1.231	1.286

1843seals24	1696	753	68	1	14	803	819	1.231	1.286
1843seals25	1368	704	40	1	12	730	742	1.091	1.167
1843seals26	1517	679	44	1	15	706	721	1.071	1.133
1843seals27	4195	1516	179	1	24	1665	1693	1.217	1.250
1843seals28	1515	586	70	1	15	636	654	1.286	1.333
1856kelle01	25625	5516	1399	1	59	6840	6913	1.254	1.266
1866raabe01	13045	3004	691	1	45	3640	3693	1.201	1.219
1877meyer01	1523	801	56	1	14	840	855	1.154	1.214
1877meyer02	573	331	26	1	8	347	355	1.143	1.250
1877meyer03	1052	551	46	1	11	583	595	1.200	1.273
1877meyer04	2550	1142	79	1	18	1197	1219	1.294	1.333
1877meyer05	1249	658	47	1	12	690	703	1.182	1.250
1877meyer06	833	471	34	1	10	492	503	1.222	1.300
1877meyer07	1229	652	47	1	13	683	697	1.167	1.231
1877meyer08	1028	556	43	1	11	585	597	1.200	1.273
1877meyer09	776	441	40	1	9	471	479	1.000	1.111
1877meyer10	940	493	41	1	11	520	532	1.200	1.273
1877meyer11	2398	1079	88	1	17	1146	1165	1.188	1.235
1888storm01	38306	6233	1292	1	76	7427	7523	1.275	1.285
1891busch01	15820	4642	527	1	44	5112	5167	1.284	1.300
1899schni01	2793	961	109	1	19	1044	1068	1.329	1.365
1903wedek01	4035	1336	122	1	26	1428	1456	1.120	1.154
1903wedek02	6040	1731	179	1	31	1872	1908	1.200	1.226
1903wedek03	7402	1934	276	1	34	2168	2208	1.212	1.235
1903wedek04	1297	646	44	1	13	676	688	1.000	1.077
1910loens01	1672	706	95	1	15	782	799	1.214	1.267
1910loens02	2988	928	141	1	23	1042	1067	1.136	1.174
1910loens03	4063	1162	172	1	26	1303	1332	1.160	1.192
1910loens04	3713	1081	167	1	24	1218	1246	1.217	1.250
1910loens05	4676	1235	254	1	28	1457	1487	1.111	1.143
1910loens06	4833	1364	244	1	29	1573	1606	1.179	1.207
1910loens07	7743	1862	414	1	36	2232	2274	1.200	1.222
1910loens08	6093	1724	328	1	31	2015	2050	1.167	1.194
1910loens09	9252	2126	453	1	39	2531	2577	1.211	1.231
1910loens10	6546	1736	274	1	35	1968	2008	1.176	1.200
1910loens11	4102	1294	217	1	27	1481	1509	1.077	1.111
1910loens12	4432	1318	221	1	26	1507	1537	1.200	1.231
1910loens13	1361	556	60	1	14	600	614	1.077	1.143
1917suder01	11437	2427	507	1	43	2879	2932	1.260	1.277
1919kafka01	10256	2321	448	1	41	2717	2767	1.248	1.266
1931tucho01	8544	2449	351	1	35	2757	2798	1.195	1.218

1931tucho02	7106	1935	207	1	35	2100	2140	1.164	1.187
1931tucho03	9699	2502	336	1	38	2790	2836	1.232	1.252
1931tucho04	7415	1968	214	1	35	2139	2180	1.208	1.231
1931tucho05	4823	1399	174	1	28	1537	1571	1.242	1.269
2001pseud01	728	363	30	1	10	381	391	1.111	1.200
2001pseud02	612	326	23	1	9	339	347	1.000	1.111
2001rieder01	1161	510	36	1	12	532	544	1.091	1.167
2001rieder02	1231	472	55	1	13	511	525	1.167	1.231

5.4.10. Semantic diversification of prefixes and suffixes. Unfortunately, we have data only from four languages at our disposal, namely German: Rothe (1989), Altmann, Best, Kind (1987); Middle High German: Kaliuščenko (1988); Slovak: Nemcová (1991, 2007) and Hungarian: Beöthy, Altmann (1984a,b). The results are presented in Table 5.16.

.Table 5.16
Computing the *c* and *p* coefficients for prefixes and suffixes
$\overline{p}$ = 1.174, s_p = 0.026, $\overline{c}$ = 1.386, s_c = 0.167

Affixes	R	$f(1)$	$f(R)$	h	L	L_{max}	p	c
Hungarian *föl-*	10	11	3	4.00	13.77	17	1.077	1.308
Hunarian *el-*	9	83	1	3.00	86.92	90	1.540	1.693
Hungarian *be-*	13	20	1	5.00	25.83	31	1.293	1.434
Hungarian *ki-*	11	12	2	4.75	15.87	20	1.101	1.291
Hungarian *meg-*	9	107	1	5.00	110.23	114	0.942	1.154
German *ab-*	11	16	1	3.00	22.01	25	1.495	1.663
German *aus-*	12	24	1	5.00	29.23	34	1.193	1.354
German *be-*	16	86	1	5.25	94.63	100	1.264	1.404
German *ein-*	9	18	1	3.50	22.28	25	1.088	1.349
German *ent-*	6	71	1	3.00	72.30	75	1.350	1.567
German *ver-*	13	42	1	6.25	46.03	53	1.328	1.435
Middle High German *be-*	13	36	1	3.00	44.26	47	1.370	1.580
Middle High German *ent-*	5	46	1	3.00	46.08	49	1.460	1.640
Middle High German *ver-*	15	14	1	3.50	23.45	27	1.420	1.586
Slovak *do-*	4	22	3	3.25	19.43	22	1.142	1.406
Slovak *na-*	6	30	7	4.86	23.81	28	1.085	1.274
Slovak *o-*	5	17	4	4.50	13.88	17	0.891	1.138

Slovak *ob-*	3	19	4	2.60	15.17	17	1.144	1.473
Slovak *od-*	6	25	4	2.75	23.61	26	1.366	1.596
Slovak *po-*	5	59	18	4.43	41.30	45	1.079	1.287
Slovak *pre-*	4	33	7	3.57	26.27	29	1.062	1.325
Slovak *pri-*	3	26	7	2.87	19.13	21	1.000	1.348
Slovak *roz-*	4	26	22	3.25	5.58	7	0.631	1.052
Slovak *s-/z-*	6	71	5	4.78	66.50	71	1.190	1.360
Slovak *u-*	4	56	2	3.91	54.13	57	0.986	1.246
Slovak *vy-*	5	61	23	4.60	38.54	42	0.961	1.187
Slovak *za-*	4	77	8	3.79	69.09	72	1.043	1.296
German *–os/ös*	9	59	2	4.50	60.16	65	1.383	1.520
German *–al/-ell*	16	93	1	9.90	96.78	107	1.148	1.234

5.4.11. Meaning diversification of English words. The testing of other aspects (entities) continues in Table 5.17 with the meaning diversification of English words as recently investigated by Fan, Popescu, Altmann (2008).

Table 5.17
Computing the *c* and *p* coefficients for meaning diversification of 165 English words (in this table the maximum rank *R* means the number *S* of different word senses/meanings)
$\bar{p}$ = 1.189, s_p = 0.194, $\bar{c}$ = 1.456, s_c = 0.181

Word	N	S	$f(1)$	$f(S)$	h	L	L_{max}	p	c
Animal	69	3	67	1	1.98	67.008	68	1.013	1.511
Ash	5	4	2	1	1.50	3.414	4	1.172	1.724
Back	302	28	92	1	8.66	109.189	118	1.150	1.248
Bad	72	17	51	1	3.00	63.839	66	1.081	1.387
Bark	12	9	4	1	1.75	10.162	11	1.117	1.622
Belly	14	6	8	1	2.00	10.497	12	1.503	1.752
Bird	36	6	31	1	1.92	34.017	35	1.069	1.554
Bite	25	13	12	1	2.00	21.464	23	1.536	1.768
Black	91	23	56	1	4.00	74.252	77	0.916	1.187
Blood	677	6	637	1	3.91	637.353	641	1.253	1.444
Blow	72	29	25	1	4.33	47.921	52	1.225	1.404
Bone	17	6	10	1	2.33	12.307	14	1.273	1.585
Breast	12	6	6	1	2.00	8.537	10	1.463	1.731
Breathe	33	9	25	1	1.94	31.021	32	1.042	1.536
Burn	55	20	11	1	4.25	26.090	29	0.895	1.155
Child	823	4	625	3	3.86	622.087	625	1.019	1.273

Cloud	51	13	24	1	2.87	33.096	35	1.018	1.360
Cold	75	16	40	1	4.20	51.204	54	0.874	1.142
Come	792	22	275	1	7.67	286.154	295	1.326	1.414
Correct	40	12	15	1	4.25	22.046	25	0.909	1.166
Count	52	11	23	1	4.00	28.436	32	1.188	1.391
Cut	2138	71	1672	1	10.74	1728.055	1741	1.329	1.392
Day	1314	10	648	1	7.25	648.648	656	1.176	1.290
Die	160	14	142	1	2.67	152.103	154	1.136	1.460
Dig	23	11	9	1	2.60	16.246	18	1.096	1.444
Dirty	25	13	12	1	2.00	21.464	23	1.536	1.768
Dog	50	8	42	1	2.00	46.427	48	1.573	1.787
Drink	74	10	32	1	4.00	35.825	40	1.392	1.544
Dry	59	19	20	1	3.73	34.526	37	0.906	1.199
Dull	30	19	5	1	2.75	20.576	22	0.813	1.245
Dust	64	7	42	1	3.00	44.296	47	1.352	1.568
Ear	51	5	36	1	3.50	36.530	39	0.988	1.277
Earth	105	9	57	1	3.94	61.279	64	0.926	1.198
Eat	680	6	479	1	5.17	478.223	483	1.145	1.311
Egg	23	5	19	1	1.92	21.028	22	1.057	1.548
Eye	291	6	264	1	4.33	264.499	268	1.051	1.270
Fall	169	44	46	1	5.33	81.896	88	1.410	1.520
Far	155	10	62	1	5.00	64.793	70	1.302	1.441
Fat	34	10	22	1	2.00	28.439	30	1.561	1.780
Father	86	9	72	1	2.66	76.658	79	1.411	1.632
Fear	127	8	73	1	4.43	75.293	79	1.081	1.288
Feather	12	7	6	1	1.83	10.099	11	1.086	1.585
Fight	729	9	268	5	7.50	263.857	271	1.099	1.219
Fire	1017	17	616	1	9.25	620.869	631	1.228	1.311
Fish	22	6	14	1	2.50	15.874	18	1.417	1.650
Float	33	15	14	1	3.00	24.700	27	1.150	1.433
Flow	66	14	18	1	5.00	25.117	30	1.221	1.377
Flower	41	4	31	1	2.75	31.093	33	1.090	1.421
Fly	74	20	33	1	4.34	46.662	51	1.299	1.460
Fog	25	4	18	1	2.67	17.686	20	1.386	1.616
Foot	1282	14	740	1	6.00	745.230	752	1.354	1.462
Forest	39	3	36	1	2.00	35.429	37	1.571	1.786
Freeze	26	14	7	1	3.00	16.708	19	1.146	1.431
Fruit	16	5	10	1	2.00	11.476	13	1.524	1.762
Full	94	13	42	1	3.80	49.021	53	1.421	1.573

Give	805	42	181	1	13.33	206.697	221	1.160	1.223
Good	303	27	190	1	7.50	207.529	215	1.149	1.263
Grass	50	10	41	1	1.96	48.012	49	1.029	1.524
Green	44	14	26	1	2.67	36.124	38	1.123	1.452
Guts	8	6	2	1	2.00	5.414	6	0.586	1.293
Hair	64	6	59	1	1.93	62.009	63	1.066	1.550
Hand	265	16	216	1	4.00	225.684	230	1.439	1.579
Head	337	42	208	1	6.00	241.748	248	1.250	1.375
Hear	356	5	275	1	4.50	274.207	278	1.084	1.287
Heart	88	10	42	1	4.25	45.760	50	1.305	1.468
Hit	1627	24	440	1	12.00	449.554	462	1.131	1.204
Hold	3906	45	1134	1	19.00	1154.762	1177	1.235	1.276
Horn	19	11	7	1	2.33	14.359	16	1.234	1.563
Hunt	19	15	4	1	2.00	15.650	17	1.350	1.675
Husband	71	2	70	1	1.83	69.010	70	1.193	1.634
Ice	40	10	31	1	1.95	38.017	39	1.035	1.530
Kill	121	17	103	1	2.33	116.241	118	1.323	1.613
Knee	55	3	51	1	2.33	50.246	52	1.318	1.611
Know	968	12	593	1	6.40	597.174	603	1.079	1.223
Lake	5	3	3	1	1.67	3.236	4	1.140	1.655
Laugh	83	4	65	1	2.87	65.043	67	1.046	1.379
Leaf	25	6	20	1	1.91	23.026	24	1.070	1.557
Left	485	24	151	1	11.43	161.431	173	1.109	1.187
Leg	90	9	75	1	2.84	79.521	82	1.348	1.577
Lie	208	10	89	1	6.50	91.241	97	1.047	1.194
Live	264	19	133	1	6.00	144.634	150	1.073	1.228
Liver	15	5	11	1	1.91	13.050	14	1.044	1.545
Louse	5	4	2	1	1.50	3.414	4	1.172	1.724
Man	2283	13	1437	1	6.00	1441.570	1448	1.286	1.405
Meat	6	3	4	1	1.75	4.162	5	1.117	1.622
Moon	38	9	30	1	1.91	36.017	37	1.080	1.562
Mother	107	7	100	1	2.00	103.419	105	1.581	1.790
Mountain	18	2	17	1	1.97	16.030	17	1.000	1.508
Mouth	74	11	49	1	3.00	54.897	58	1.551	1.701
Name	847	15	698	1	6.00	703.697	711	1.461	1.551
Near	80	9	44	1	3.73	48.132	51	1.051	1.305
Neck	38	5	34	1	1.91	36.015	37	1.082	1.563
New	1648	12	980	1	6.00	982.995	990	1.401	1.501
Night	1041	8	736	1	6.29	735.799	742	1.172	1.304

Nose	45	14	30	1	2.33	40.255	42	1.312	1.607
Old	1066	9	515	1	4.60	516.965	522	1.399	1.529
Play	331	52	70	1	8.67	109.342	120	1.390	1.460
Pull	90	24	44	1	5.00	62.241	66	0.940	1.152
Push	88	15	56	1	4.00	65.311	69	1.230	1.422
Rain	44	4	25	1	3.40	24.227	27	1.155	1.404
Red	79	8	43	1	5.40	45.202	49	0.863	1.074
Right	1032	35	649	1	11.75	670.492	682	1.071	1.150
Road	99	4	95	1	2.00	95.420	97	1.580	1.790
Root	29	15	11	1	2.50	21.891	24	1.406	1.644
Rope	8	4	5	1	1.80	6.123	7	1.096	1.598
Round	48	26	13	1	3.00	34.062	37	1.469	1.646
Rub	23	5	19	1	1.91	21.028	22	1.068	1.556
Salt	39	10	26	1	2.60	32.147	34	1.158	1.482
Sand	14	4	10	1	2.00	10.476	12	1.524	1.762
Say	3547	12	2593	1	8.20	2593.961	2603	1.255	1.346
Scratch	29	14	9	1	2.75	19.485	21	0.866	1.278
Sea	43	4	38	1	2.33	38.250	40	1.316	1.609
See	1227	25	617	1	12.00	626.970	640	1.185	1.252
Seed	28	13	12	1	2.60	21.194	23	1.129	1.464
Sew	6	2	5	1	1.80	4.120	5	1.100	1.600
Sharp	45	15	9	1	4.50	19.153	22	0.813	1.077
Sing	86	5	46	1	3.00	46.297	49	1.351	1.568
Sit	187	8	134	1	4.33	136.249	140	1.126	1.328
Skin	28	11	11	1	3.00	17.891	20	1.055	1.370
Sky	50	2	49	1	1.83	48.010	49	1.193	1.634
Sleep	85	6	58	1	2.91	60.037	62	1.028	1.362
Smell	46	8	14	1	4.50	15.994	20	1.145	1.335
Smoke	91	10	68	1	3.25	73.233	76	1.230	1.467
Smooth	30	12	11	1	3.00	18.482	21	1.259	1.506
Snake	29	8	20	1	2.33	24.265	26	1.304	1.603
Snow	37	6	13	1	3.70	14.706	17	0.850	1.161
Spit	18	8	11	1	1.91	16.050	17	1.044	1.545
Split	31	19	5	1	3.00	19.657	22	1.172	1.448
Squeeze	34	17	10	1	3.25	22.521	25	1.102	1.378
Stand	330	24	169	1	6.60	183.892	191	1.269	1.380
Star	25	12	8	1	3.00	16.335	18	0.832	1.222
Stick	48	26	7	1	4.24	28.405	31	0.801	1.084
Stone	629	16	330	1	5.00	338.612	344	1.347	1.478

Straight	59	21	14	1	5.00	28.690	33	1.077	1.262
Suck	10	6	4	1	2.00	6.650	8	1.350	1.675
Sun	65	7	47	1	2.84	50.056	52	1.056	1.389
Swell	24	11	5	1	3.50	11.657	14	0.937	1.241
Swim	14	3	12	1	1.91	12.045	13	1.049	1.547
Tail	17	11	6	1	2.00	13.537	15	1.463	1.731
Think	602	14	277	1	4.80	283.383	289	1.478	1.587
Throw	110	20	53	1	5.50	66.140	71	1.080	1.247
Tie	47	18	13	1	4.00	25.719	29	1.094	1.320
Tongue	27	10	14	1	2.67	19.700	22	1.377	1.610
Tooth	43	5	38	1	2.00	39.428	41	1.572	1.786
Tree	113	7	107	1	1.91	111.005	112	1.094	1.568
Turn	2091	38	744	1	12.25	765.494	780	1.289	1.347
Walk	1208	17	1092	1	6.75	1099.549	1107	1.296	1.400
Warm	57	13	34	1	3.00	42.354	45	1.323	1.549
Wash	56	21	13	1	4.50	27.764	32	1.210	1.386
Water	1026	10	744	1	3.98	747.843	752	1.395	1.547
Wet	34	9	23	1	2.50	28.189	30	1.208	1.525
White	117	25	65	1	4.33	84.331	88	1.102	1.309
Wind	47	15	29	1	2.50	39.848	42	1.435	1.661
Wing	31	10	8	1	4.25	13.227	16	0.853	1.123
Wipe	18	2	17	1	1.93	16.030	17	1.043	1.539
Woman	587	4	480	5	4.00	475.243	478	0.919	1.189
Worm	8	5	3	1	2.00	4.828	6	1.172	1.586
Year	865	4	832	1	3.40	831.146	834	1.189	1.428
Yellow	42	8	26	1	2.86	29.525	32	1.331	1.565
Fingernail	1	1	1	1	1.00	no length			
River	55	1	55	55	1.00	no length			
Rotten	3	3	1	1	1.00	$L = L_{max}$			
Stab	6	6	1	1	1.00	$L = L_{max}$			
Vomit	4	4	1	1	1.00	$L = L_{max}$			
Wife	120	1	120	120	1.00	no length			

The extreme distributions of the last 6 words (out of 165) cannot be worked out since either there is no arc length (that is $S = 1$), or $L = L_{max}$ and $h = 1$ lead to the indeterminacy of $p = (L_{max} - L)/(h - 1)$.

The results show that there is a kind of "prescribed" development of meaning diversification. Nevertheless, a number of problems remain open and must be scrutinized in future research: (1) Does this relationship hold only for English or can it be found in other languages, too? (2) The parameters c and p are re-

presented by their means; however, their values differs for word-frequency distributions and meaning diversifications. In diversification they are slightly greater. The question is, why? Can we conjecture that the more concrete the linguistic level, the smaller are *c and p*? That is, is there an increase in *c* or *p* beginning from phonemes, to syllables, words, morphemes, morpheme classes (e.g. prepositions), meanings, meaning classes (e.g. colours, grammatical categories)? How can *c* or *p* be interpreted? Are they associated with redundancy or other requirements that must be fulfilled by language (cf. Köhler 2005)?

5.4.12. Word forms of 100 texts in 20 languages. This is the usual evaluation of word forms which are easily available in any written language. Since here 20 languages are taken into account, the evaluation has a greater weight. The values for individual texts are given in Table 5.18 and the linear relationship between *c* and *p* is presented in Figure 5.9. From the interlinguistic textological point of view this result is the most important one.

Table 5.18
Computing the *c* and *p* coefficients for word forms for 100 texts in 20 languages
$\overline{p}$ = 1.223, s_p = 0.132, $\overline{c}$ = 1.292, s_c = 0.116
(in this table the vocabulary *V* means the maximum rank *R*)

ID	*N*	*V*	*f*(1)	*f*(*V*)	*h*	*L*	L_{max}	*p*	*c*
B 01	761	400	40	1	10	428	438	1.111	1.200
B 02	352	201	13	1	8	205	212	1.000	1.125
B 03	515	285	15	1	9	290	298	1.000	1.111
B 04	483	286	21	1	8	297	305	1.143	1.250
B 05	406	238	19	1	7	247	255	1.333	1.429
Cz 01	1044	638	58	1	9	684	694	1.250	1.333
Cz 02	984	543	56	1	11	586	597	1.100	1.182
Cz 03	2858	1274	182	1	19	1432	1454	1.222	1.263
Cz 04	522	323	27	1	7	342	348	1.000	1.143
Cz 05	999	556	84	1	9	627	638	1.375	1.444
E 01	2330	939	126	1	16	1043	1063	1.333	1.375
E 02	2971	1017	168	1	22	1157	1183	1.238	1.273
E 03	3247	1001	229	1	19	1205	1228	1.278	1.316
E 04	4622	1232	366	1	23	1567	1596	1.318	1.348
E 05	4760	1495	297	1	26	1761	1790	1.160	1.192
E 07	5004	1597	237	1	25	1801	1832	1.292	1.320
E 13	11265	1659	780	1	41	2388	2437	1.225	1.244
G 05	559	332	30	1	8	351	360	1.286	1.375
G 09	653	379	30	1	9	398	407	1.125	1.222
G 10	480	301	18	1	7	310	317	1.167	1.286

G 11	468	297	18	1	7	307	313	1.000	1.143
G 12	251	169	14	1	6	175	181	1.200	1.333
G 14	184	129	10	1	5	133	137	1.000	1.200
G 17	225	124	11	1	6	128	133	1.000	1.167
H 01	2044	1079	225	1	12	1289	1302	1.182	1.250
H 02	1288	789	130	1	8	907	917	1.429	1.500
H 03	403	291	48	1	4	332	337	1.667	1.750
H 04	936	609	76	1	7	674	683	1.500	1.571
H 05	413	290	32	1	6	314	320	1.200	1.333
Hw 03	3507	521	277	1	26	764	796	1.280	1.308
Hw 04	7892	744	535	1	38	1229	1277	1.297	1.316
Hw 05	7620	680	416	1	38	1047	1094	1.270	1.289
Hw 06	12356	1039	901	1	44	1877	1938	1.419	1.432
I 01	11760	3667	388	1	37	4007	4053	1.278	1.297
I 02	6064	2203	257	1	25	2426	2458	1.333	1.360
I 03	854	483	64	1	10	534	545	1.222	1.300
I 04	3258	1237	118	1	21	1330	1353	1.150	1.190
I 05	1129	512	42	1	12	537	552	1.364	1.417
In 01	376	221	16	1	6	228	235	1.400	1.500
In 02	373	209	18	1	7	219	225	1.000	1.143
In 03	347	194	14	1	6	200	206	1.200	1.333
In 04	343	213	11	1	5	217	222	1.250	1.400
In 05	414	188	16	1	8	196	202	0.857	1.000
Kn 003	3188	1833	74	1	13	1891	1905	1.167	1.231
Kn 004	1050	720	23	1	7	733	741	1.333	1.429
Kn 005	4869	2477	101	1	16	2558	2576	1.200	1.250
Kn 006	5231	2433	74	1	20	2481	2505	1.263	1.300
Kn 011	4541	2516	63	1	17	2558	2577	1.188	1.235
Lk 01	345	174	20	1	8	185	192	1.000	1.125
Lk 02	1633	479	124	1	17	580	601	1.313	1.353
Lk 03	809	272	62	1	12	318	332	1.273	1.333
Lk 04	219	116	18	1	6	126	132	1.200	1.333
Lt 01	3311	2211	133	1	12	2328	2342	1.273	1.333
Lt 02	4010	2334	190	1	18	2502	2522	1.176	1.222
Lt 03	4931	2703	103	1	19	2783	2804	1.167	1.211
Lt 04	4285	1910	99	1	20	1983	2007	1.263	1.300
Lt 05	1354	909	33	1	8	930	940	1.429	1.500
Lt 06	829	609	19	1	7	621	626	0.833	1.000
M 01	2062	398	152	1	18	527	548	1.235	1.278
M 02	1175	277	127	1	15	386	402	1.143	1.200

M 03	1434	277	128	1	17	385	403	1.125	1.176
M 04	1289	326	137	1	15	444	461	1.214	1.267
M 05	3620	514	234	1	26	715	746	1.240	1.269
Mq 01	2330	289	247	1	22	507	534	1.286	1.318
Mq 02	457	150	42	1	10	179	190	1.222	1.300
Mq 03	1509	301	218	1	14	500	517	1.308	1.357
Mr 001	2998	1555	75	1	14	1612	1628	1.231	1.286
Mr 018	4062	1788	126	1	20	1890	1912	1.158	1.200
Mr 026	4146	2038	84	1	19	2099	2120	1.167	1.211
Mr 027	4128	1400	92	1	21	1468	1490	1.100	1.143
Mr 288	4060	2079	84	1	17	2141	2161	1.250	1.294
R 01	1738	843	62	1	14	886	903	1.308	1.357
R 02	2279	1179	110	1	16	1269	1287	1.200	1.250
R 03	1264	719	65	1	12	770	782	1.091	1.167
R 04	1284	729	49	1	10	764	776	1.333	1.400
R 05	1032	567	46	1	11	599	611	1.200	1.273
R 06	695	432	30	1	10	452	460	0.889	1.000
Rt 01	968	223	111	1	14	316	332	1.231	1.286
Rt 02	845	214	69	1	13	265	281	1.333	1.385
Rt 03	892	207	66	1	13	256	271	1.250	1.308
Rt 04	625	181	49	1	11	216	228	1.200	1.273
Rt 05	1059	197	74	1	15	251	269	1.286	1.333
Ru 01	753	422	31	1	8	441	451	1.429	1.500
Ru 02	2595	1240	138	1	16	1357	1376	1.267	1.313
Ru 03	3853	1792	144	1	21	1909	1934	1.250	1.286
Ru 04	6025	2536	228	1	25	2732	2762	1.250	1.280
Ru 05	17205	6073	701	1	41	6722	6772	1.250	1.268
Sl 01	756	457	47	1	9	494	502	1.000	1.111
Sl 02	1371	603	66	1	13	651	667	1.333	1.385
Sl 03	1966	907	102	1	13	991	1007	1.333	1.385
Sl 04	3491	1102	328	1	21	1404	1428	1.200	1.238
Sl 05	5588	2223	193	1	25	2385	2414	1.208	1.240
Sm 01	1487	267	159	1	17	403	424	1.313	1.353
Sm 02	1171	222	103	1	15	304	323	1.357	1.400
Sm 03	617	140	45	1	13	168	183	1.250	1.308
Sm 04	736	153	78	1	12	214	229	1.364	1.417
Sm 05	447	124	39	1	11	149	161	1.200	1.273
T 01	1551	611	89	1	14	681	698	1.308	1.357
T 02	1827	720	107	1	15	807	825	1.286	1.333
T 03	2054	645	128	1	19	749	771	1.222	1.263

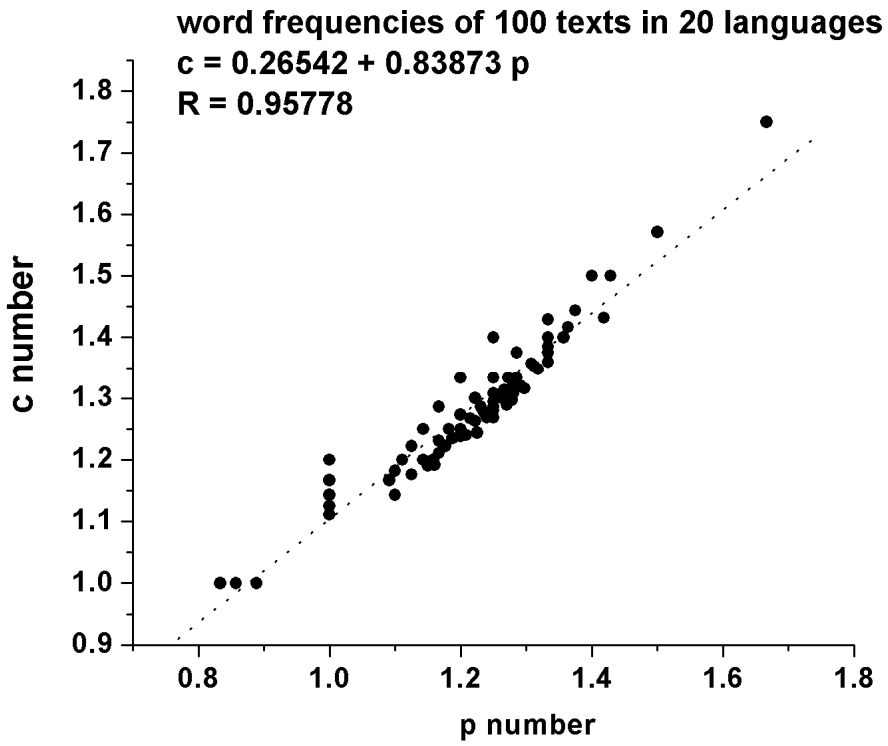


Figure 5.9. Dependence of *c* on *p*

From two projects which are in statu nascendi we obtained the permission to present the indicator *p* in 12 Slavic languages (E. Kelih) yielding for word form frequencies $\overline{p}$ = 1.24 with standard deviation 0.07, and for the lemmas in 60 annual speeches of Italian presidents (A. Tuzzi) $\overline{p}$ = 1.28, standard deviation 0.10. Lemmas which are a higher abstraction than forms have a slightly greter *p*.

5.4.13. Diversification of word meanings in French. In addition to the semantic diversification of word meanings in English (Fan, Popescu, Altmann 2008) there is a well scrutinized case of French *et* in *Le petit prince* by A. Saint-Exupéry by U. Rothe (1986), who found 70 senses and the distribution is given in Table 5.19

Table 5.19
Computing the *c* and *p* coefficients for the diversification of the French *et* (Rothe 1986)
$\overline{p}$ = 1.234, $\overline{c}$ = 1.362

	R	*f(1)*	*f(R)*	*h*	*L*	L_{max}	*p*	*c*
French *et* (Rothe 1986)	70	17	1	6	78.83	85	1.234	1.362

Adding this result to the 165 cases of English word diversification of Table 5.17 the means $\overline{p}$ and $\overline{c}$ do not change..

5.4.14. Grammatical categories. There is only one case of diversification known, namely Rothe (1991a) in which the functions and the meanings of the German genitive are analyzed. The data are given in Table 5.20.

Table 5.20
Computing the *c* and *p* coefficients for the German genitive
$\overline{p}$ = 1.259, $\overline{c}$ = 1.335.

Category	*R*	*f(1)*	*f(R)*	*h*	*L*	L_{max}	*p*	*c*
German genitive	55	47	1	9.80	88.92	100	1.259	1.335

5.4.15. Word associations. Word associations are a still deeper penetration in the meaning of words which can be considered as composed of (possibly unique) denotation, a number of connotations which can be even public and a great number of associations which are private and individual, though intersection is possible. In order to analyze at least one language, we used the count of French associations collected by Thérouanne and Denhière (2004). and analyzed by Nemcová, Popescu, Altmann (2009) where one can find the complete results concernig 162 French words. The mean $\overline{p}$ = 1.262 and its standard deviation is 0.150, the mean $\overline{c}$ = 1.422 and its standard deviation is 0.136.

5.5. Conclusions on language levels

First of all, we must admit that the set of phenomena presented above does not allow us to set up founded hypotheses; in the best case we can make some conjectures. Neither the number of languages nor the diversity of language phenomena is sufficient. Diversification is often taken into account in grammar without regard to the frequency of phenomena. Grammarians content themselves with listing the existing cases; in language didactics frequency plays a more important role, but it is used only implicitly. Looking at Table 5.21 and 5.22 where the mean coefficients $\overline{p}$ and $\overline{c}$ are presented, one could conjecture that both increase from class building encompassing a complete field of phenomena to diversification of individual entities. The strongest diversification is that of meaning of independent full words followed by that of modifying affixes. The fact that in some languages affixes do not exist does not change anything. Affixes can diversify only if they exist.

Tables 5.21 and 5.22 show that if one takes into account class diversification, i.e. a classification comprising a complete inventory such as that of letters, word classes (parts of speech), a lexical word class or possible rhythmic

units, the coefficient *p* and *c* are small. Phonic phenomena are on the lower end of the scale, semantic phenomena at the upper end. Morphological phenomena are somewhere in the middle. If one performed a different classification of parts of speech, e.g. with a stronger emphasis on syntax, it could be expected that the coefficients *p* and *c* would increase. On the other hand, diversifications taking into account whole classes (e.g. inventories) yield smaller *p* or *c* than diversification of individual entities (e.g. word meanings).

The end of this examination is open. We did not take into account any boundary conditions and only a small number of (available) phenomena. Nevertheless, two facts can be observed: cross-linguistic constancy of *p* and *c* for the same phenomenon and different magnitudes for different phenomena. A scaling of linguistic phenomena would be premature. It would be also necessary to make experiments with the data, namely to remove an element from a class and observe the change of *p* or *c*. If this criterion is "valid", then it could make the classification (class forming) of linguistic phenomena more objective.

Table 5.21
The mean $\overline{p}$ and its interval (ranked by $\overline{p}$)

Category	$\overline{p}$	s_p	*p*-interval	$\overline{p}$-interval
1. Sounds, phonemes and letters	1.01	0.03	<0.96, 1.06>	<1.00, 1.02>
2. Word classes (parts of speech)	1.02	0.09	<0.85, 1.20>	<0.96, 1.09>
3. Rhythmic patterns (Latin, Greek, German)	1.06	0.11	<0.84, 1.28>	<1.02, 1.10>
4. Pitches of 58 musical texts	1.09	0.10	<0.88, 1.29>	<1.06, 1.11>
5. Colour classes	1.09	0.07	<0.94,1.23>	<1.07, 1.10>
6. Allomorphs of German plural	1.11	0.22	<0.68, 1.53>	<1.04, 1.17>
7. Polish paradigmatic classes	1.11	0.05	<1.01, 1.21<	1.06, 1.16>
8. Auxiliaries	1.13	0.14	<0.88, 1.39>	<1.04, 1.22>
9. Word frequencies for German	1.17	0.10	<0.97, 1.36>	<1.15, 1.18>
10. Affixes (meaning diversification)	1.17	0.03	<1.12, 1.23>	<1.16, 1.18>
11. Words (meaning diversification)	1.19	0.19	0.81, 1.57>	<1.16, 1.22>
12. Word frequencies for 20 languages	1.22	0.13	<0.96, 1.48>	<1.20, 1.25>
13. French *et*	1.23	-	-	-
14. Word frequencies in 12 Slavic languages (same text)	1.24	0.07	<1.10, 1.38>	<1.21, 1.25>
15. German genitive	1.26	-	-	-
16. French associations	1.26	0.15	<0.94, 1.56>	<1.24, 1.28>
17. Lemmas in 60 Italian texts	1.28	0.10	<1.08, 1.48>	<1.25, 1.31>

Table 5.22
The mean $\overline{c}$ and its interval (ranked by $\overline{c}$)

Category	$\overline{c}$	s_c	c-interval	$\overline{c}$-interval
1. Sounds, phonemes, letters	1.05	0.03	<1.00, 1.10>	<1.04, 1.06>
2. Pitches of 58 musical texts	1.13	0.10	<0.94, 1.32>	<1.11,1.16>
3. Word classes (parts of speech)	1.14	0.08	<0.98, 1.30>	<1.08, 1.20>
4. Rhythmic patterns (Latin, Greek, German)	1.14	0.11	<0.92, 1.36>	<1.10, 1.18>
5. Polish paradigmatic classes	1.15	0.05	<1.05, 1.25>	<1.10, 1.20>
6. Colour classes	1.19	0.07	<1.05, 1.32>	<1.14, 1.23>
7. Word frequencies for German	1.22	0.09	<1.06, 1.39>	<1.21, 1.24>
8. Auxiliaries	1.24	0.11	<1.03, 1.45>	<1.17, 1.32>
9. Word frequencies for 20 languages	1.29	0.12	<1.06. 1.52>	<1.27, 1.31>
10. German genitive	1.34	-	-	-
11. Allomorphs of plural	1.35	0.20	<0.97, 1.74>	<1.29, 1.41>
12. French *et*	1.36	-	-	-
13. Affixes (Meaning diversification)	1.39	0.17	<1.06, 1.71>	<1.33, 1,45>
14. French associations	1.42	0.14	<1.16, 1.69>	<1.41, 1.44>
15. Words (Meaning diversification)	1.46	0.18	<1.10, 1.81>	<1.43, 1.48>

A graphic presentation of the means and their intervals can be seen in Figures 5.10 and 5.11.

If p (or c) is really a text constant expressing a constant relation between the properties of the rank-frequency sequence of word forms, then this phenomenon is lawlike and the special form of the monotonous decrease of frequencies seems to abide by an unknown control mechanism. It is true that we always obtain *p ± boundary conditions* and *c ± boundary conditions* whose finding is a task for philologists. The decrease of frequencies is not always strictly monotonous – because some frequencies occur several times, especially in the tail of the sequence – but a principally concave sequence will probably never be found. The limit of convexity of these sequences is L_{min}, as defined above.

Both from linguistic and from textological point of view, the problem seems to become a stream flowing into a broad delta of boundary conditions and ending in the fuzzy ocean of linguistic states. Looking at Figures 5.10 and 5.11 we may ask whether there is an asymptote for diversification, or are there some attractors for individual language levels and phenomena

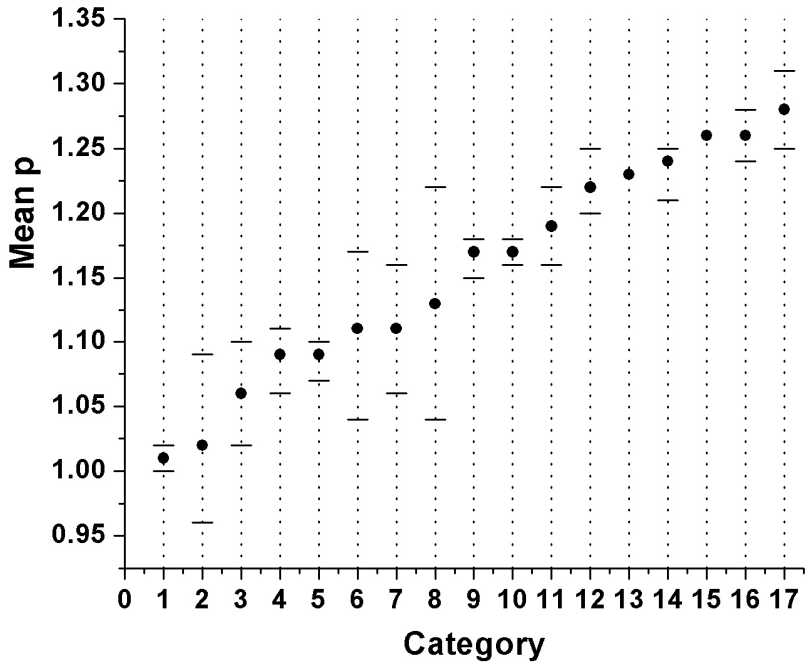


Figure 5.10. The positioning of linguistic categories regarding the mean $\overline{p}$

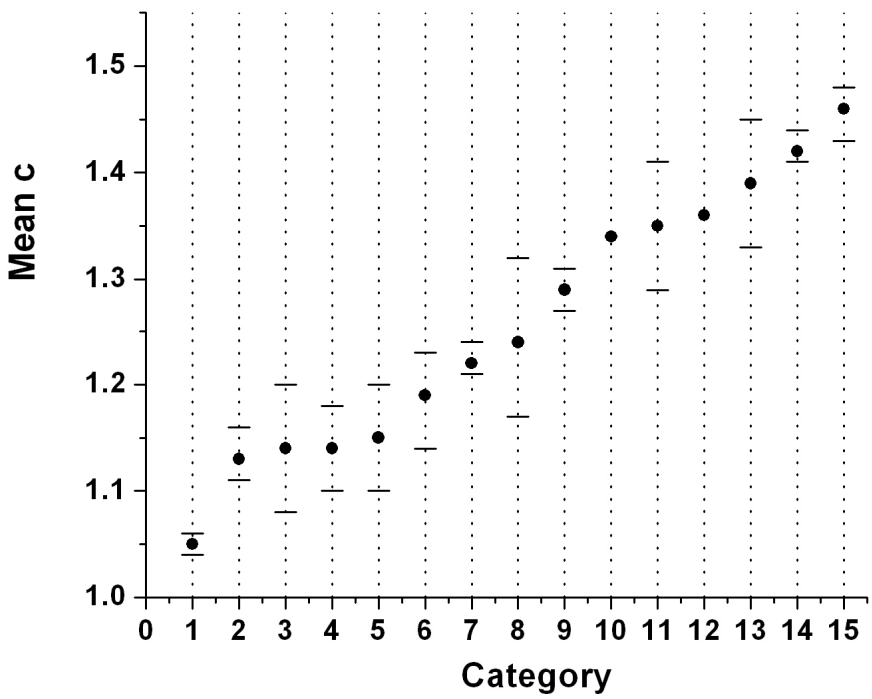


Figure 5.11. The positioning of linguistic categories regarding the mean $\overline{c}$

6. Hapax legomena

Hapax legomena are those words that occur in text only once. In the rank-frequency sequence they occupy the highest ranks and form the tail of the sequence. Since we study word forms, many of them belong to lemmas occurring several times but the given form may be unique. This fact automatically leads to the consequence that more synthetic languages have more hapax legomena than analytic ones. Strongly analytic languages have a smaller number of forms, hence words have a greater chance to be repeated.

The number of hapax legomena is sometimes used as a measure of vocabulary richness, but if word forms are not lemmatized, it is rather the form-richness of the language it characterizes. Further, it is evident that up to a certain text length the number of hapax legomena increases but later on it may begin to decrease because in long texts even forms may be repeated. Last but not least, why only hapax legomena should contribute to vocabulary richness? The whole vocabulary of the text excluding auxiliaries is a feature of richness. A more adequate richness indicator using the post-*h* region has been proposed in Popescu et al. (2009).

Hence, in homogeneous texts of moderate length, hapax legomena are rather indicators of the position of a given language on the synthetism-analytism scale (cf. Popescu, Altmann 1008a; Popescu, Altmann, Köhler 2008; Popescu, Mačutek, Altmann 2008). It has been shown empirically that the longer the arc L, the more hapax legomena there are in the text. The relation is linear, as can be seen in Figure 6.1. The dependence is

$$HL = 0.6571L - 28.6704$$

yielding $R^2 = 0.9719$.

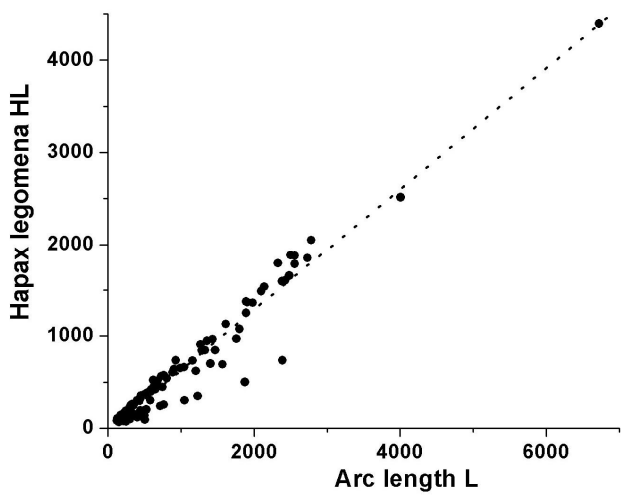


Figure 6.1. Dependence of hapax legomena on arc length for 100 texts in 20 languages

Thus a further indicator akin to those defined above can be introduced in form

$$(6.1) \quad B_5 = \frac{Hapax\ Legomena\ HL}{Arc\ Length\ L},$$

theoretically positioned in the range <0,1>. The corresponding individual text values of B_5 are given in the fourth column of Table 6.1. The names of languages can be found in Table 5.1.

Table 6.1
Dependence of hapax legomena on arc length

ID	*L*	*HL*	$B_5 = HL/L$	*ID*	*L*	*HL*	$B_5 = HL/L$
B 01	428.45	298	0.696	Lk 03	317.63	174	0.548
B 02	205.38	153	0.745	Lk 04	125.56	80	0.637
B 03	289.80	212	0.732	Lt 01	2328.00	1792	0.770
B 04	297.03	222	0.747	Lt 02	2502.00	1878	0.751
B 05	247.30	187	0.756	Lt 03	2783.00	2049	0.736
Cz 01	684.17	517	0.756	Lt 04	1983.00	1359	0.685
Cz 02	586.22	412	0.703	Lt 05	930.00	737	0.792
Cz 03	1432.06	964	0.673	Lt 06	621.00	521	0.839
Cz 04	341.99	241	0.705	M 01	526.92	202	0.383
Cz 05	626.98	445	0.710	M 02	386.01	146	0.378
E 01	1042.85	662	0.635	M 03	384.62	133	0.346
E 02	1157.22	735	0.635	M 04	444.29	192	0.432
E 03	1204.91	620	0.515	M 05	715.18	239	0.334
E 04	1567.31	693	0.442	Mq 01	506.98	91	0.179
E 05	1760.86	971	0.551	Mq 02	178.59	86	0.482
E 07	1800.70	1075	0.597	Mq 03	500.37	138	0.276
E 13	2388.47	736	0.308	Mr 001	1612.43	1128	0.700
G 05	351.41	250	0.711	Mr 018	1890.34	1249	0.661
G 09	398.43	302	0.758	Mr 026	2098.93	1486	0.708
G 10	309.84	237	0.765	Mr 027	1467.65	846	0.576
G 11	306.80	232	0.756	Mr 288	2141.01	1534	0.716
G 12	175.44	141	0.804	R 01	886.35	606	0.684
G 14	132.54	107	0.807	R 02	1269.07	908	0.715
G 17	127.96	84	0.656	R 03	770.20	567	0.736
H 01	1288.83	844	0.655	R 04	764.36	573	0.750

H 02	907.18	638	0.703	R 05	599.19	424	0.708
H 03	332.44	259	0.779	R 06	451.75	353	0.781
H 04	674.06	509	0.755	Rt 01	315.91	127	0.402
H 05	314.40	250	0.795	Rt 02	264.75	128	0.483
Hw 03	764.27	255	0.334	Rt 03	255.86	98	0.383
Hw 04	1229.31	347	0.282	Rt 04	215.58	102	0.473
Hw 05	1047.48	302	0.288	Rt 05	250.69	73	0.291
Hw 06	1876.68	500	0.266	Ru 01	441.04	316	0.716
I 01	4007.01	2514	0.627	Ru 02	1356.70	946	0.697
I 02	2426.40	1604	0.661	Ru 03	1909.09	1365	0.715
I 03	534.33	382	0.715	Ru 04	2731.76	1850	0.677
I 04	1329.65	848	0.638	Ru 05	6722.04	4395	0.654
I 05	537.49	355	0.660	Sl 01	493.72	364	0.737
In 01	228.49	166	0.727	Sl 02	651.09	423	0.650
In 02	218.62	147	0.672	Sl 03	990.94	651	0.657
In 03	199.85	130	0.650	Sl 04	1404.13	701	0.499
In 04	217.37	145	0.667	Sl 05	2385.35	1593	0.668
In 05	195.65	121	0.618	Sm 01	403.17	119	0.295
Kn 003	1891.11	1373	0.726	Sm 02	303.92	96	0.316
Kn 004	733.26	564	0.769	Sm 03	168.39	75	0.445
Kn 005	2558.43	1784	0.697	Sm 04	214.17	76	0.355
Kn 006	2481.41	1655	0.667	Sm 05	149.49	66	0.442
Kn 011	2557.69	1873	0.732	T 01	680.99	465	0.683
Lk 01	184.77	127	0.687	T 02	807.46	540	0.669
Lk 02	579.97	302	0.521	T 03	748.50	447	0.597

If we take the means of B_5 for individual languages, we obtain the results in Table 6.2. Here, again, great B_5 is characteristic for highly synthetic languages. The more analytic languages are at the end of the scale.

The situation is presented graphically in Figure 6.2, where the dispersion does not seem to play a relevant role. Nevertheless, it could be taken into account in studies concerning style, genre etc.

Table 6.2
Means of B_5 for individual languages (ranked by decreasing mean B_5)

	Language	**mean B_5**			**Language**	**mean B_5**
1	Latin	0.762		11	Italian	0.660
2	German	0.751		12	Tagalog	0.650
3	Hungarian	0.738		13	Slovenian	0.642
4	Bulgarian	0.735		14	Lakota	0.598
5	Romanian	0.729		15	English	0.526
6	Kannada	0.718		16	Rarotongan	0.407
7	Czech	0.709		17	Maori	0.375
8	Russian	0.692		18	Samoan	0.371
9	Marathi	0.672		19	Marquesan	0.312
10	Indonesian	0.667		20	Hawaiian	0.293

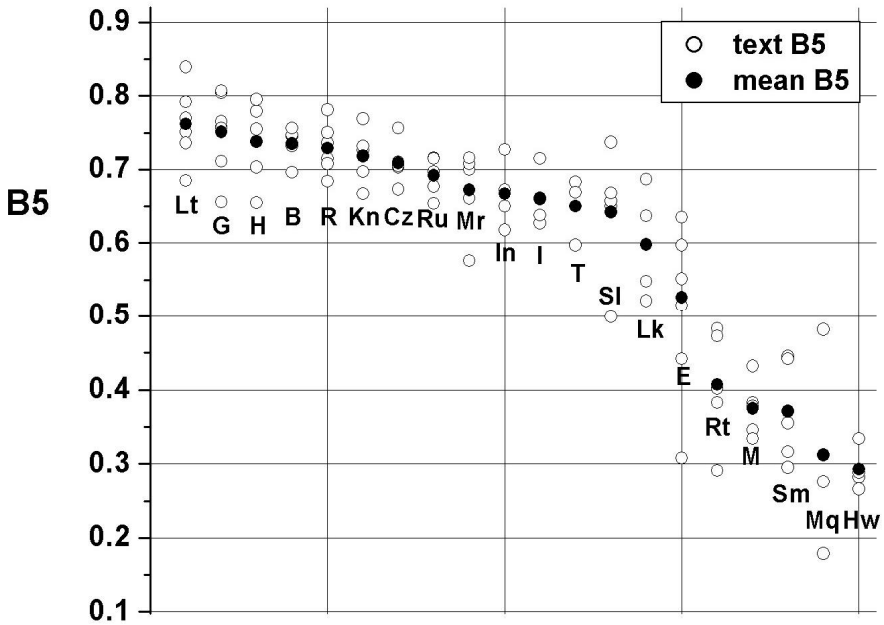


Figure 6.2. The indicator B_5 for 20 languages

The same linear relationship can be found between vocabulary (V) and hapax legomena of the text as presented in Figure 6.3. (cf. Popescu, Altmann 2008a).

The straight line is

$$HL = 0.7256V - 18.6979$$

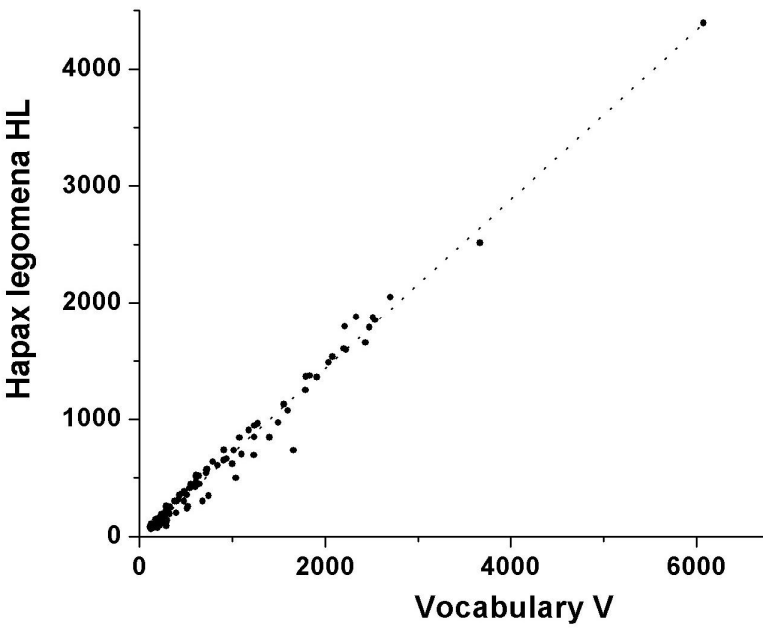


Figure 6.3. Linear dependence between vocabulary and hapax legomena for 100 texts in 20 languages

yielding R^2 = 0.9842. The very good fit can be used to obtain the variance of the index B_5 and, consequently, to test differences between indices of this type. V is considered to be a constant. We have

$$B_5 = \frac{HL}{L} \cong \frac{0.7256V - 18.6979}{L},$$

hence

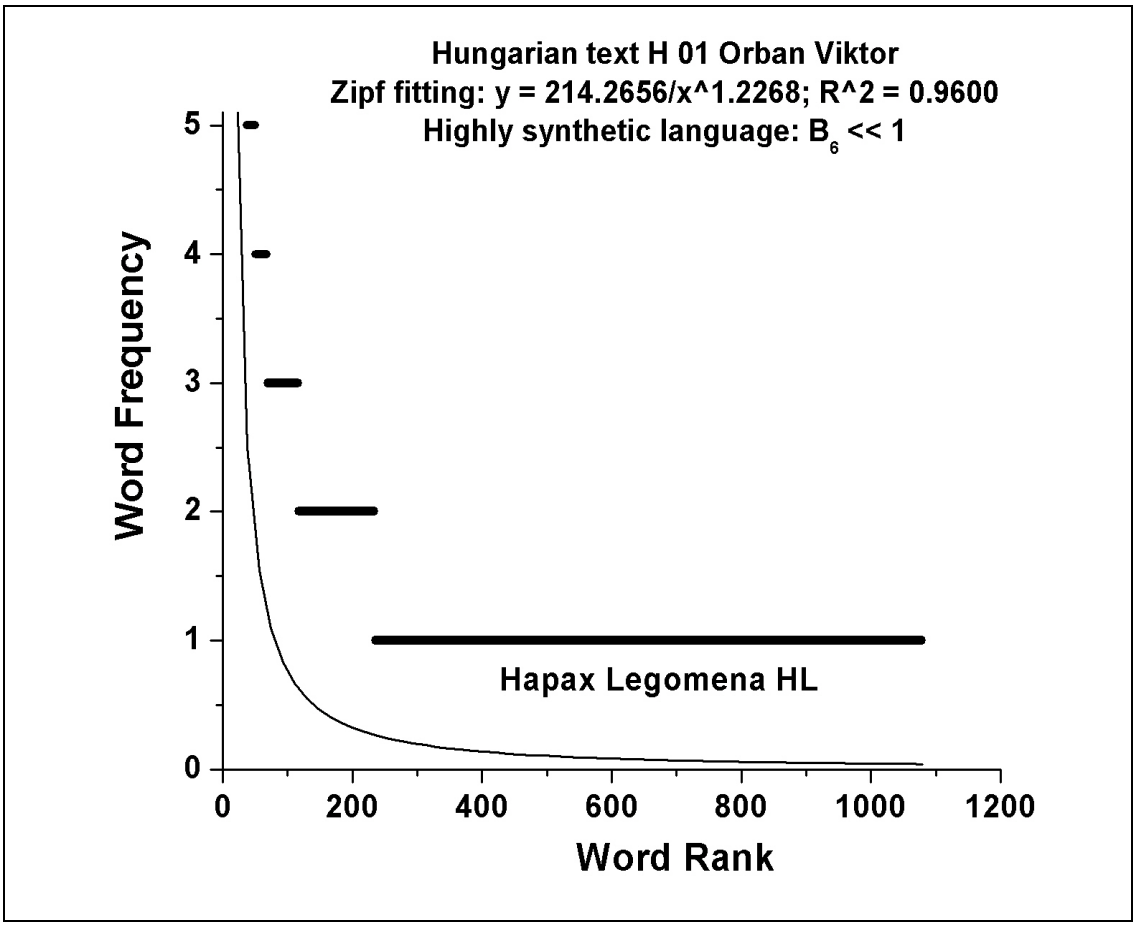
$$Var(B_5) \cong (0.7256V - 18.6979)^2 Var\left(\frac{1}{L}\right).$$

$Var\left(\frac{1}{L}\right)$ is derived in Chapter 5. The difference between two indices is significant if

$$\frac{\left|B_5 - B_5^*\right|}{\sqrt{Var(B_5) + Var(B_5^*)}} > 1.96 .$$

The approximation can be used only if a text is not too short (based on the analysis of 100 texts from 20 languages, $V = 100$ seems to be enough).

Though the difference between languages using B_5 is well visible in Fig. 6.2, this indicator has a rather small empirical range and testing the difference of two languages is a rather complex procedure. In order to show visually the status of hapax legomena we fit the original Zipfian function $f(r) = c/r^a$ to some data. As can be expected, in a highly analytic language with small number of forms, the Zipfian function will lie over the hapax legomena tail while in a highly synthetic language it will lie under the hapax legomena tail. In the first case the function overestimates the tail length, in the latter it underestimates it. In a moderately analytic/synthetic language the function will cross the hapax legomena tail somewhere in the middle, i.e. approximately in HL/2. The situation can be seen in Figure 6.4.



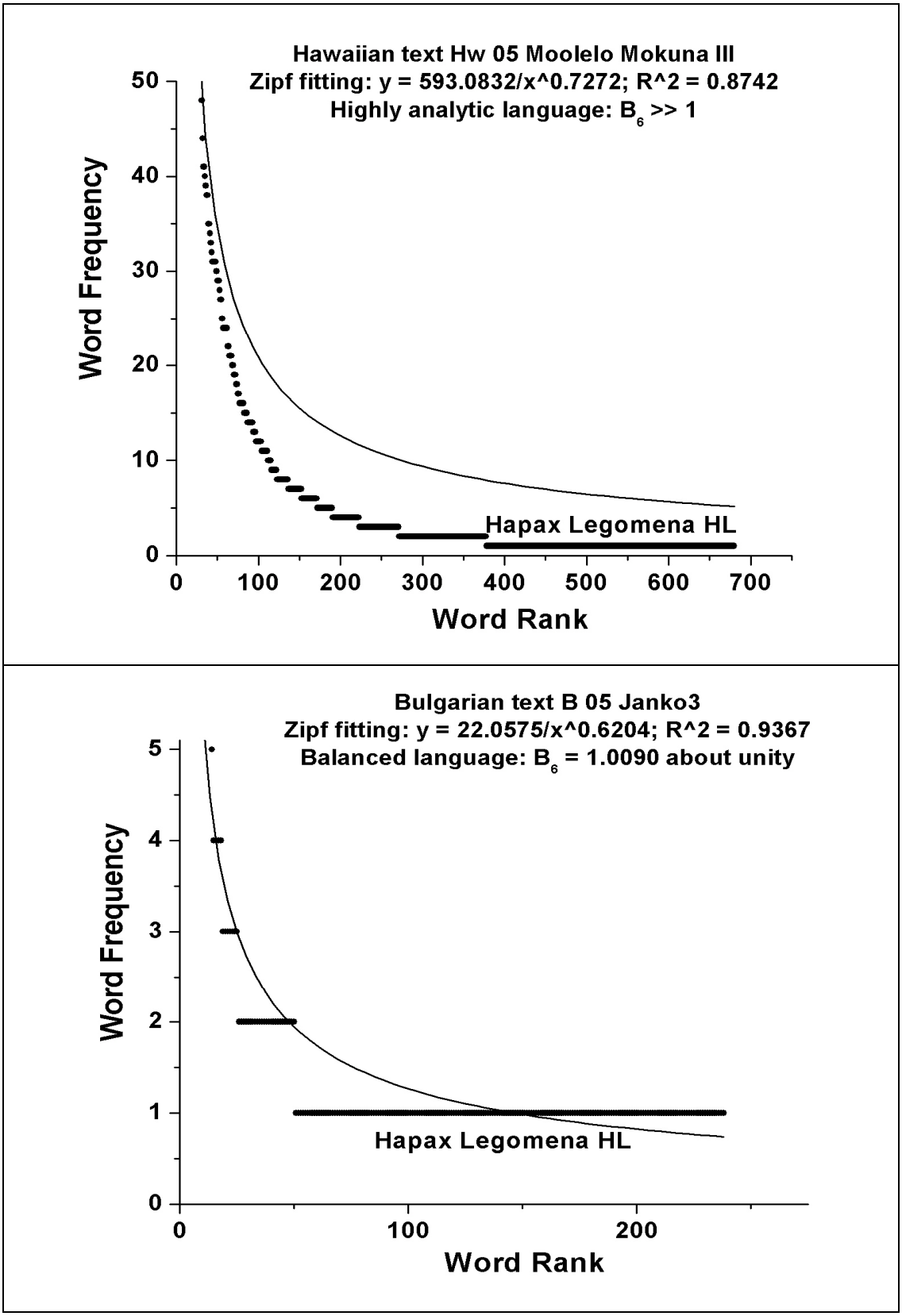


Figure 6.4. The relation of the Zipfian function to the hapax level in different languages: highly synthetic (top), highly analytic (middle), balanced (bottom).

Using this fact we can define another indicator in which we replace *r* of the Zipfian function by $r = V - HL/2$, i.e. by the point at which in balanced languages the Zipfian curve crosses the level of hapax legomena in the mid point. Thus we obtain

$$(6.2) \quad B_6 = \frac{c}{(V - HL/2)^a},$$

where *a* and *c* are the parameters of the Zipf function and *V* is the size of vocabulary (cf. Popescu, Altmann 2008a). In Table 6.3 the fitting of the Zipfian function to 100 texts, the determination coefficient signalizing very good fits, and the computation of B_6 is shown.

Table 6.3
Fitting Zipf's function to data of 100 texts from 20 languages and the indicator B_6

ID	*V*	Zipf *a*	Zipf *c*	R^2	*HL*	$B_6 = \frac{\mathbf{c}}{\mathbf{(V - HL / 2)^a}}$
B 01	400	0.6850	41.8602	0.9837	298	0.9507
B 02	201	0.5704	17.6950	0.8705	153	1.1292
B 03	285	0.5550	20.9975	0.8790	212	1.1798
B 04	286	0.6169	23.6917	0.9619	222	0.9790
B 05	238	0.6202	22.0499	0.9367	187	1.0090
Cz 01	638	0.7473	54.2844	0.9764	517	0.6416
Cz 02	543	0.7169	51.9648	0.9767	412	0.8013
Cz 03	1274	0.8028	175.4805	0.9832	964	0.8261
Cz 04	323	0.6228	23.3822	0.9537	241	0.8562
Cz 05	556	0.8722	77.1944	0.9715	445	0.4864
E 01:	939	0.7657	145.9980	0.9620	662	1.0783
E 02:	1017	0.7434	180.1325	0.9661	735	1.4610
E 03:	1001	0.8179	254.7482	0.9752	620	1.2123
E 04:	1232	0.8712	385.9532	0.9870	693	1.0449
E 05:	1495	0.8009	319.1386	0.9822	971	1.2529
E 07	1597	0.7568	300.1258	0.9347	1075	1.5416
E 13	1659	0.8034	811.1689	0.9800	736	2.5688
G 05	332	0.6935	32.8211	0.9646	250	0.8129
G 09	379	0.6523	32.5565	0.9626	302	0.9431
G 10	301	0.6053	21.8114	0.9402	237	0.9331
G 11	297	0.5895	19.9677	0.9593	232	0.9320
G 12	169	0.6062	14.3627	0.9514	141	0.8888
G 14	129	0.5755	10.8110	0.9349	107	0.8977

G 17	124	0.5515	13.1021	0.9349	84	1.1531
H 01	1079	1.2268	214.2708	0.9600	844	0.0749
H 02	789	1.1865	122.0057	0.9365	638	0.0824
H 03	291	1.2114	44.9653	0.8864	259	0.0950
H 04	609	0.9549	74.8581	0.9451	509	0.2753
H 05	290	0.8168	30.9795	0.9093	250	0.4784
Hw 03	521	0.7932	329.6012	0.9489	255	2.8821
Hw 04	744	0.7633	678.1305	0.9154	347	5.3384
Hw 05	680	0.7267	592.6243	0.8742	302	6.2199
Hw 06	1039	0.7816	1081.7823	0.9352	500	5.8855
I 01	3667	0.7266	509.5979	0.9336	2514	1.7784
I 02	2203	0.7488	305.6487	0.9559	1604	1.3468
I 03	483	0.7895	56.8099	0.9523	382	0.6427
I 04	1237	0.7014	153.3448	0.9385	848	1.3948
I 05	512	0.6524	54.5840	0.9293	355	1.2306
In 01	221	0.5809	18.2346	0.9486	166	1.0420
In 02	209	0.5915	19.1717	0.9583	147	1.0509
In 03	194	0.5417	15.6229	0.9565	130	1.1233
In 04	213	0.4877	11.9156	0.9574	145	1.0683
In 05	188	0.5374	19.4218	0.8843	121	1.4347
Kn 003	1833	0.6072	66.4545	0.9775	1373	0.9223
Kn 004	720	0.5237	22.1001	0.9699	564	0.9144
Kn 005	2477	0.6621	124.5588	0.9105	1784	0.9480
Kn 006	2433	0.5809	95.9573	0.9522	1655	1.3181
Kn 011	2516	0.5786	77.0267	0.9666	1873	1.0862
Lk 01	174	0.6416	23.4838	0.9348	127	1.1474
Lk 02	479	0.7731	139.2126	0.9510	302	1.5798
Lk 03	272	0.7512	71.8668	0.9527	174	1.4240
Lk 04	116	0.6792	18.7509	0.9801	80	0.9901
Lt 01	2211	0.7935	109.3668	0.9078	1792	0.3666
Lt 02	2334	0.8047	160.3530	0.9335	1878	0.4729
Lt 03	2703	0.6366	109.5291	0.9832	2049	0.9695
Lt 04	1910	0.6505	129.2023	0.9463	1359	1.2627
Lt 05	909	0.5877	34.1056	0.9713	737	0.8449
Lt 06	609	0.5293	19.3370	0.9325	521	0.8726
M 01	398	0.7680	185.4091	0.9225	202	2.3386
M 02	277	0.8197	123.4636	0.9693	146	1.5787
M 03	277	0.7902	147.8281	0.9557	133	2.1571
M 04	326	0.8353	137.7184	0.9763	192	1.4664
M 05	514	0.7484	297.2460	0.9306	239	3.3897
Mq 01	289	0.8030	240.0615	0.9588	91	2.9102
Mq 02	150	0.7440	46.4870	0.9655	86	1.4370

Mq 03	301	0.9795	225.2046	0.9856	138	1.0853
Mr 001	1555	0.6293	78.3965	0.9815	1128	1.0210
Mr 018	1788	0.6685	128.5531	0.9863	1249	1.1470
Mr 026	2038	0.6224	101.6971	0.9633	1486	1.1758
Mr 027	1400	0.6166	120.0829	0.9456	846	1.7214
Mr 288	2079	0.6304	100.2890	0.9683	1534	1.0857
R 01	843	0.6720	73.6423	0.9571	606	1.0739
R 02	1179	0.7567	115.8007	0.9802	908	0.7930
R 03	719	0.7175	60.8094	0.9778	567	0.7771
R 04	729	0.6673	52.4236	0.9798	573	0.8993
R 05	567	0.6746	48.1009	0.9743	424	0.9157
R 06	432	0.6349	30.3691	0.9350	353	0.8995
Rt 01	223	0.8575	123.9533	0.9645	127	1.6008
Rt 02	214	0.7469	83.2271	0.9316	128	1.9726
Rt 03	207	0.7208	78.6409	0.9465	98	2.0454
Rt 04	181	0.7359	60.2092	0.9358	102	1.6749
Rt 05	197	0.6917	87.0541	0.9469	73	2.5959
Ru 01	422	0.6538	36.1404	0.9604	316	0.9437
Ru 02	1240	0.7713	138.5450	0.9915	946	0.8251
Ru 03	1792	0.7106	158.2659	0.9620	1365	1.0851
Ru 04	2536	0.7181	234.3457	0.9571	1850	1.1661
Ru 05	6073	0.7826	775.3826	0.9807	4395	1.2063
Sl 01	457	0.7467	44.1840	0.9760	364	0.6665
Sl 02	603	0.6846	68.9001	0.9823	423	1.1571
Sl 03	907	0.7685	115.2402	0.9604	651	0.8651
Sl 04	1102	0.9187	334.8100	0.9912	701	0.7633
Sl 05	2223	0.7232	240.2785	0.9490	1593	1.2572
Sm 01	267	0.8285	177.1858	0.9678	119	2.1315
Sm 02	222	0.7752	123.5355	0.9450	96	2.2641
Sm 03	140	0.6858	58.1896	0.8708	75	2.4320
Sm 04	153	0.7925	89.0771	0.9563	76	2.0738
Sm 05	124	0.7161	46.3093	0.9263	66	1.8312
T 01	611	0.7624	120.0367	0.8817	465	1.2995
T 02	720	0.7803	144.5780	0.8685	540	1.2297
T 03	645	0.7652	167.7334	0.8923	447	1.6447

The position of individual languages can be easily estimated if one computes the means of B_6. This can be seen in Table 6.4 where the languages are ordered according to increasing B_6. As can be seen, highly synthetic languages have very small values while highly analytic languages attain $B_6 > 5$. Maybe some Polynesian languages attain still more extreme values but the problem must be solved

simultaneously both theoretically (by mathematicians) and empirically (by specialists for Polynesian languages). We restrict ourselves to a one-dimensional graphical presentation of the position of individual languages on the analytism/ synthetism scale (cf. Figure 6.5)

Table 6.4
Mean analytism indicator B_6 of 20 languages
(ranked by increasing mean B_6)

	Language	Mean B_6	Number of texts
1	Hungarian	0.2012	5
2	Czech	0.7223	5
3	Latin	0.7982	6
4	Romanian	0.8931	6
5	German	0.9372	7
6	Slovenian	0.9418	5
7	Kannada	1.0378	5
8	Russian	1.0453	5
9	Bulgarian	1.0495	5
10	Indonesian	1.1438	5
11	Marathi	1.2302	5
12	Italian	1.2787	5
13	Lakota	1.2853	4
14	Tagalog	1.3913	3
15	English	1.4514	7
16	Marquesan	1.8108	3
17	Rarotongan	1.9779	5
18	Samoan	2.1465	5
19	Maori	2.1861	5
20	Hawaiian	5.0815	4

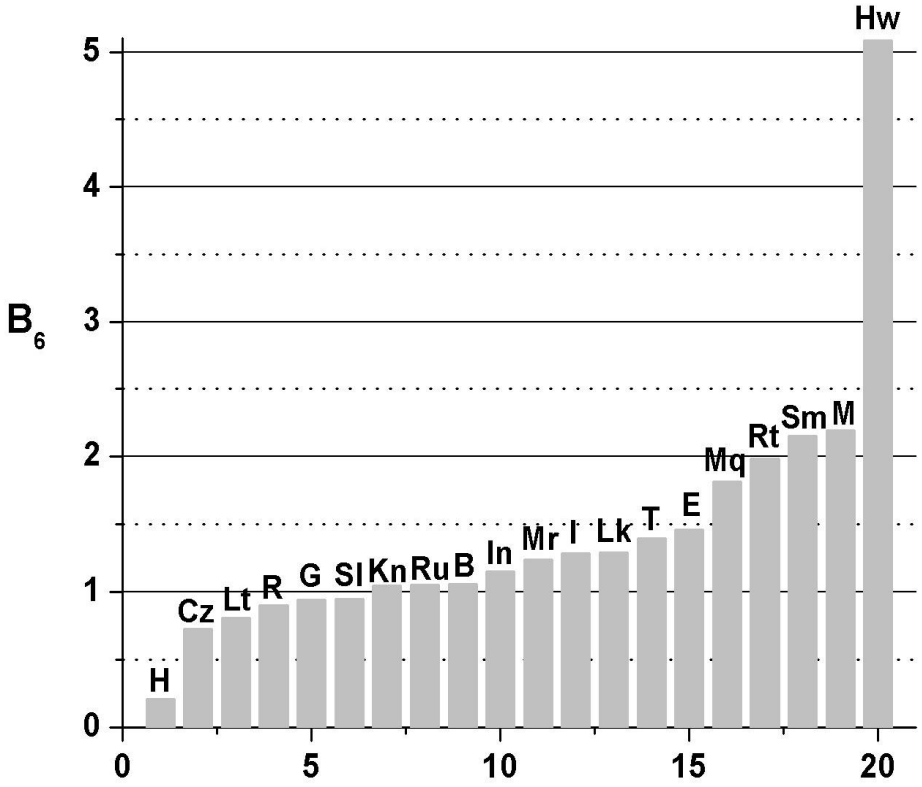


Figure 6.5. The analytic character of some languages as revealed by B_6

7. Further typological considerations

Though for fitting purposes we abandoned Zipf´s (zeta) function and replaced it by a sum of exponentials, the function demonstrated its utility in typological considerations. But the typology of languages is an analogy to text typology in individual languages, e.g. associated with style, genre etc., hence all results of typological studies of a language can be transferred into the study of texts in a given language. This task can be left to specialized philologists; here it is dealt with in a general way.

As has been shown in Chapter 6, for highly analytic languages Zipf´s function lies over the hapax legomena; the contrary holds for highly synthetic languages. In the first case it means that the empirical average defined as

$$(7.1) \quad M_E = \frac{1}{N}\sum_{r=1}^{V} rf(r),$$

where N is text length, V is the vocabulary, r the rank and $f(r)$ the frequency at rank r, must be much smaller than the average M_Z computed using the Zipfian $f(r) = c/r^a$ inserted in (7.1). The parameters c and a must be computed iteratively. On the contrary, in highly synthetic languages the empirical mean will be greater than the Zipfian. In order to express the difference, we define the indicator (cf. Popescu, Altmann 2008b)

$$(7.2) \quad B_7 = \frac{M_E - M_Z}{M_E},$$

which can be interpreted as follows:

if $B_7 > 0$, then the language tends to synthetism
if $B_7 < 0$, then the language tends to analytism
if $B_7 \approx 0$, then the language is balanced and contains both kinds of phenomena.

As an example let us consider the frequency count of word forms in the Hawaiian text Hw 05 Moolelo Mokuna III (also presented in Figure 6.4). The empirical text size and mean yield $N = 7620$ and $M_E = 68.7388$. Now, using the Zipfian iterative fitting $f(r) = c/r^a$, we obtain the function $f(r) = 592.6243r^{-0.7267}$. Its text size (the integral of $f(r)$ from 1 to V) and mean yield $N_Z = 11056.8403$ and $M_Z = 170.3493$. Inserting these two means in (7.2) we obtain

$$B_7\,(Hw\ 05) = \frac{68.7388 - 170.3493}{68.7388} = -1.4782,$$

showing the well known fact that Hawaiian is a rather analytic language. The indicator B_7 gives at the same time the degree of analytism.

Since in all cases the decisive circumstance is the height of the fitting Zipfian function in point V representing the highest rank, for characterisation purposes it is sufficient to consider the indicator

$$(7.3) \quad B_8 = \frac{c}{V^a},$$

i.e. the value of the Zipfian end point.

Finally, perhaps the most transparent Zipfian typological indicator is the arithmetic mean

$$(7.4) \quad B_9 = \overline{\left(\frac{f - f_Z}{f}\right)}$$

of the relative deviation of the observed frequencies from the Zipfian fitting function $f_Z = c/r^a$. In order to compare these indicators, we present them in Table 7.1 together with the hapax legomena controlled index B_6 and other necessary numbers.

Table 7.1
Indicators B_6, B_7, B_8, B_9, from 100 texts in 20 languages

ID	***V***	**Zipf *a***	**Zipf *c***	M_E	M_Z	B_6	B_7	B_8	B_9
B 01	400	0.6850	41.8602	116.4139	109.6275	0.9507	0.0583	0.6909	-0.0299
B 02	201	0.5704	17.6950	63.1108	65.6908	1.1292	-0.0409	0.8593	-0.1643
B 03	285	0.5550	20.9975	87.5379	93.6461	1.1798	-0.0698	0.9114	-0.1929
B 04	286	0.6169	23.6917	91.5569	87.2274	0.9790	0.0473	0.7230	-0.0548
B 05	238	0.6202	22.0499	75.3153	72.8480	1.0090	0.0328	0.7405	-0.0903
Cz 01	638	0.7473	54.2844	205.6006	154.7747	0.6416	0.2472	0.4352	0.2196
Cz 02	543	0.7169	51.9648	162.8963	139.7022	0.8013	0.1424	0.5692	0.0881
Cz 03	1274	0.8028	175.4805	311.3947	268.0846	0.8261	0.1391	0.5640	0.0266
Cz 04	323	0.6228	23.3822	108.8831	97.3214	0.8562	0.1062	0.6401	0.0694
Cz 05	556	0.8722	77.1944	164.7137	107.4763	0.4864	0.3475	0.3114	0.3659
E 01	939	0.7657	145.9980	216.7004	216.0852	1.0783	0.0028	0.7730	-0.1793
E 02	1017	0.7434	180.1325	202.6156	242.9598	1.4610	-0.1991	1.0468	-0.5358
E 03	1001	0.8179	254.7482	192.9960	207.0470	1.2123	-0.0728	0.8953	-0.2658
E 04	1232	0.8712	385.9532	223.1696	223.4339	1.0449	-0.0012	0.7836	-0.0544
E 05	1495	0.8009	319.1386	286.4662	313.1640	1.2529	-0.0932	0.9148	-0.3144
E 07	1597	0.7568	300.1258	303.6303	364.9494	1.5416	-0.2020	1.1301	-0.6074

E 13	1659	0.8034	811.1689	219.5143	343.8041	2.5688	-0.5662	2.1000	-1.0509
G 05	332	0.6935	32.8211	105.5599	90.6857	0.8129	0.1409	0.5858	0.0508
G 09	379	0.6523	32.5565	117.9433	109.0793	0.9431	0.0752	0.6770	-0.0424
G 10	301	0.6053	21.8114	100.7583	92.9696	0.9331	0.0773	0.6893	-0.0212
G 11	297	0.5895	19.9677	100.9872	93.5783	0.9320	0.0734	0.6960	-0.0072
G 12	169	0.6062	14.3627	59.9203	53.4282	0.8888	0.1083	0.6408	0.0083
G 14	129	0.5755	10.8110	47.5543	42.7453	0.8977	0.1011	0.6595	0.0064
G 17	124	0.5515	13.1021	39.8311	42.2041	1.1531	-0.0596	0.9179	-0.1202
H 01	1079	1.2268	214.2708	304.7397	69.6929	0.0749	0.7713	0.0407	0.8530
H 02	789	1.1865	122.0057	253.4014	63.2871	0.0824	0.7502	0.0446	0.8316
H 03	291	1.2114	44.9653	107.2308	28.3793	0.0950	0.7353	0.0466	0.7819
H 04	609	0.9549	74.8581	205.1592	97.1793	0.2753	0.5263	0.1642	0.5699
H 05	290	0.8168	30.9795	104.7337	65.8429	0.4784	0.3713	0.3018	0.3383
Hw 03	521	0.7932	329.6012	69.9367	117.9251	2.8821	-0.6862	2.3069	-1.2982
Hw 04	744	0.7633	678.1305	75.0495	174.0335	5.3384	-1.3189	4.3590	-2.9092
Hw 05	680	0.7267	592.6243	68.7388	170.3493	6.2199	-1.4782	5.1825	-3.4011
Hw 06	1039	0.7816	1081.7823	91.9140	230.7216	5.8855	-1.5102	4.7463	-3.4306
I 01	3667	0.7266	509.5979	677.9826	865.2727	1.7784	-0.2762	1.3109	-0.8215
I 02	2203	0.7488	305.6487	457.5523	505.2243	1.3468	-0.1042	0.9596	-0.4423
I 03	483	0.7895	56.8099	146.0597	110.6116	0.6427	0.2427	0.4320	0.2166
I 04	1237	0.7014	153.3448	275.2637	315.9784	1.3948	-0.1479	1.0391	-0.4309
I 05	512	0.6524	54.5840	134.0469	145.6400	1.2306	-0.0865	0.9322	-0.2531
In 01	221	0.5809	18.2346	71.4973	71.1092	1.0420	0.0054	0.7926	-0.0812
In 02	209	0.5915	19.1717	66.3995	66.5723	1.0509	-0.0026	0.8132	-0.0677
In 03	194	0.5417	15.6229	62.7781	65.5138	1.1233	-0.0436	0.9005	-0.0958
In 04	213	0.4877	11.9156	74.8338	75.8346	1.0683	-0.0134	0.8721	-0.0496
In 05	188	0.5374	19.4218	53.3671	63.8473	1.4347	-0.1964	1.1645	-0.2954
Kn 003	1833	0.6072	66.4545	576.1998	539.1967	0.9223	0.0642	0.6936	0.0221
Kn 004	720	0.5237	22.1001	261.3076	240.2214	0.9144	0.0807	0.7048	0.0199
Kn 005	2477	0.6621	124.5588	705.5287	664.2990	0.9480	0.0584	0.7054	-0.0068
Kn 006	2433	0.5809	95.9573	657.8180	740.4287	1.3181	-0.1256	1.0353	-0.2853
Kn 011	2516	0.5786	77.0267	764.0881	767.8495	1.0862	-0.0049	0.8297	-0.1275
Lk 01	174	0.6416	23.4838	50.0667	52.6722	1.1474	-0.0520	0.8575	-0.1726
Lk 02	479	0.7731	139.2126	89.0171	112.9533	1.5798	-0.2689	1.1788	-0.5819
Lk 03	272	0.7512	71.8668	57.7355	68.9918	1.4240	-0.1950	1.0660	-0.4065
Lk 04	116	0.6792	18.7509	35.3927	34.4326	0.9901	0.0271	0.7429	-0.0377
Lt 01	2211	0.7935	109.3668	771.1130	461.7444	0.3666	0.4012	0.2427	0.5139
Lt 02	2334	0.8047	160.3530	716.6397	474.2860	0.4729	0.3382	0.3126	0.3982
Lt 03	2703	0.6366	109.5291	803.9286	754.7652	0.9695	0.0612	0.7158	-0.0526
Lt 04	1910	0.6505	129.2023	484.4184	525.0506	1.2627	-0.0839	0.9486	-0.2980

Lt 05	909	0.5877	34.1056	319.8213	278.5167	0.8449	0.1291	0.6225	0.0543
Lt 06	609	0.5293	19.3370	230.4608	202.4373	0.8726	0.1216	0.6494	0.0249
M 01	398	0.7680	185.4091	63.9248	95.7958	2.3386	-0.4986	1.8677	-0.9241
M 02	277	0.8197	123.4636	50.5234	62.8350	1.5787	-0.2437	1.2285	-0.3974
M 03	277	0.7902	147.8281	46.2162	65.9788	2.1571	-0.4276	1.7364	-0.7596
M 04	326	0.8353	137.7184	58.6804	70.9494	1.4664	-0.2091	1.0958	-0.3767
M 05	514	0.7484	297.2460	69.4287	125.8978	3.3897	-0.8133	2.7807	-1.5941
Mq 01	289	0.8030	240.0615	44.6326	67.1753	2.9102	-0.5051	2.5361	-0.9099
Mq 02	150	0.7440	46.4870	33.6324	40.1976	1.4370	-0.1952	1.1177	-0.3399
Mq 03	301	0.9795	225.2046	50.6561	50.0045	1.0853	0.0129	0.8410	-0.0086
Mr 001	1555	0.6293	78.3965	450.1638	443.8837	1.0210	0.0140	0.7690	-0.0657
Mr 018	1788	0.6685	128.5531	454.4077	477.8562	1.1470	-0.0516	0.8606	-0.1732
Mr 026	2038	0.6224	101.6971	559.1975	584.8680	1.1758	-0.0459	0.8867	-0.2175
Mr 027	1400	0.6166	120.0829	312.5678	408.1721	1.7214	-0.3059	1.3789	-0.5339
Mr 288	2079	0.6304	100.2890	588.1170	589.0420	1.0857	-0.0016	0.8122	-0.1453
R 01	843	0.6720	73.6423	228.9908	228.8815	1.0739	0.0005	0.7961	-0.1451
R 02	1179	0.7567	115.8007	328.5853	272.9949	0.7930	0.1692	0.5489	0.0621
R 03	719	0.7175	60.8094	218.4913	182.4494	0.7771	0.1650	0.5423	0.0962
R 04	729	0.6673	52.4236	222.1083	200.3455	0.8993	0.0980	0.6445	-0.0040
R 05	567	0.6746	48.1009	169.8120	155.5140	0.9157	0.0842	0.6677	-0.0166
R 06	432	0.6349	30.3691	141.4417	126.7049	0.8995	0.1042	0.6444	-0.0187
Rt 01	223	0.8575	123.9533	38.7252	48.4559	1.6008	-0.2513	1.2009	-0.4612
Rt 02	214	0.7469	83.2271	39.0686	55.5682	1.9726	-0.4223	1.5128	-0.7643
Rt 03	207	0.7208	78.6409	40.7635	55.9160	2.0454	-0.3717	1.6835	-0.6240
Rt 04	181	0.7359	60.2092	37.5232	48.3329	1.6749	-0.2881	1.3128	-0.4977
Rt 05	197	0.6917	87.0541	37.9226	55.5516	2.5959	-0.4649	2.2528	-0.8032
Ru 01	422	0.6538	36.1404	129.4329	120.6856	0.9437	0.0676	0.6945	-0.0255
Ru 02	1240	0.7713	138.5450	323.6250	278.4930	0.8251	0.1395	0.5696	0.0442
Ru 03	1792	0.7106	158.2659	454.9782	445.0264	1.0851	0.0219	0.7719	-0.2115
Ru 04	2536	0.7181	234.3457	598.9348	614.6240	1.1661	-0.0262	0.8419	-0.2628
Ru 05	6073	0.7826	775.3826	1215.6960	1249.8376	1.2063	-0.0281	0.8488	-0.3375
Sl 01	457	0.7467	44.1840	146.7963	113.0045	0.6665	0.2302	0.4561	0.1989
Sl 02	603	0.6846	68.9001	153.3246	162.5056	1.1571	-0.0599	0.8609	-0.1934
Sl 03	907	0.7685	115.2402	235.1974	207.9875	0.8651	0.1157	0.6147	0.0263
Sl 04	1102	0.9187	334.8100	213.7368	179.9701	0.7633	0.1580	0.5370	0.1338
Sl 05	2223	0.7232	240.2785	502.7643	535.6631	1.2572	-0.0654	0.9122	-0.3449
Sm 01	267	0.8285	177.1858	41.4405	59.8669	2.1315	-0.4446	1.7297	-0.7837
Sm 02	222	0.7752	123.5355	38.3578	55.1037	2.2641	-0.4366	1.8745	-0.7669
Sm 03	140	0.6858	58.1896	26.4554	40.6778	2.4320	-0.5376	1.9639	-0.9456
Sm 04	153	0.7925	89.0771	27.3927	38.2418	2.0738	-0.3961	1.6539	-0.6961

Sm 05	124	0.7161	46.3093	25.9150	34.9991	1.8312	-0.3505	1.4673	-0.5865
T 01	611	0.7624	120.0367	133.6170	144.6973	1.2995	-0.0829	0.9020	-0.4584
T 02	720	0.7803	144.5780	157.1779	163.5462	1.2297	-0.0405	0.8522	-0.4080
T 03	645	0.7652	167.7334	119.4537	151.5339	1.6447	-0.2686	1.1877	-0.7457

A comparative presentation showing the means of individual languages is presented in Table 7.2. The attained values will surely be made more precise if more texts are added. But even in this stage one can observe that for all indicators the languages occupy approximately the same position on the analytism/ synthetism scale.

Table 7.2
Means of indicators B_6, B_7, B_8, B_9, in 20 languages

Language	$\bar{B}_6$	Language	$\bar{B}_7$	Language	$\bar{B}_8$	Language	$\bar{B}_9$
Hungarian	0.2012	Hungarian	0.6309	Hungarian	0.1196	Hungarian	0.6749
Czech	0.7223	Czech	0.1965	Czech	0.5040	Czech	0.1539
Latin	0.7982	Latin	0.1612	Latin	0.5819	Latin	0.1068
Romanian	0.8931	Romanian	0.1035	Romanian	0.6407	Romanian	-0.0044
German	0.9372	Slovenian	0.0757	Slovenian	0.6762	German	-0.0179
Slovenian	0.9418	German	0.0738	German	0.6952	Slovenian	-0.0359
Kannada	10.378	Russian	0.0349	Russian	0.7453	Kannada	-0.0755
Russian	10.453	Kannada	0.0146	Bulgarian	0.785	Bulgarian	-0.1064
Bulgarian	10.495	Bulgarian	0.0055	Kannada	0.7938	Indonesian	-0.1179
Indonesian	11.438	Indonesian	-0.0501	Indonesian	0.9086	Russian	-0.1586
Marathi	12.302	Italian	-0.0744	Italian	0.9348	Marathi	-0.2271
Italian	12.787	Marathi	-0.0782	Marathi	0.9415	Lakota	-0.2997
Lakota	12.853	Lakota	-0.1222	Lakota	0.9613	Italian	-0.3462
Tagalog	13.913	Tagalog	-0.1307	Tagalog	0.9806	Marquesan	-0.4195
English	14.514	English	-0.1617	English	10.919	English	-0.4297
Marquesan	18.108	Marquesan	-0.2291	Marquesan	14.983	Tagalog	-0.5374
Rarotongan	19.779	Rarotongan	-0.3597	Rarotongan	15.926	Rarotongan	-0.6301
Samoan	21.465	Samoan	-0.4331	Samoan	17.379	Samoan	-0.7558
Maori	21.861	Maori	-0.4385	Maori	17.418	Maori	-0.8104
Hawaiian	50.815	Hawaiian	-12.484	Hawaiian	41.487	Hawaiian	-27.598

Though none of the indicators is normalized they tell the same story. But in that case they must be related in such a way that they are transformable into each other. As a matter of fact, the relations are very simple. The relationship between B_6 and B_7 can be expressed in form

(7.5) $B_7 = 0.5331(B_6^{-0.1963} - B_6^{0.6861})$

which is presented in Figure 7.1. This relationship is not linear; it has the character of a power function.

The relationship between B_6 and B_8 is evidently linear because B_8 is, as a matter of fact, B_6 without the additive constant *HL*/2. The relationship can be expressed in form (cf. Fig 7.2)

(7.6) $B_8 = 0.8408B_6 - 0.0985$.

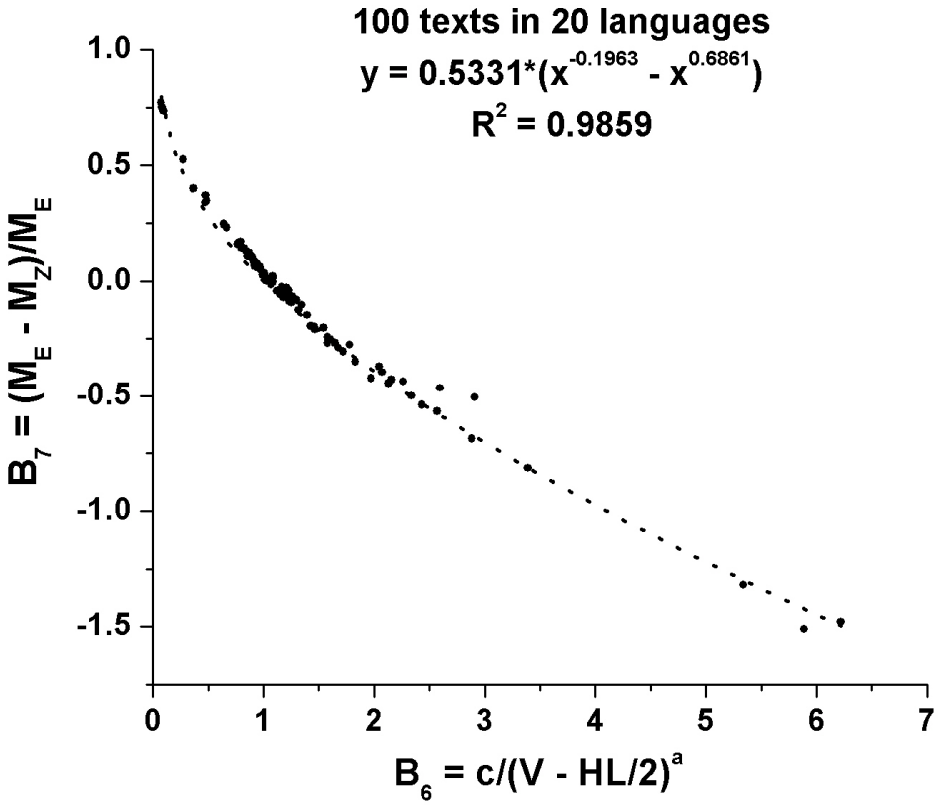


Figure 7.1. The relationship between indicators B_6 and B_7

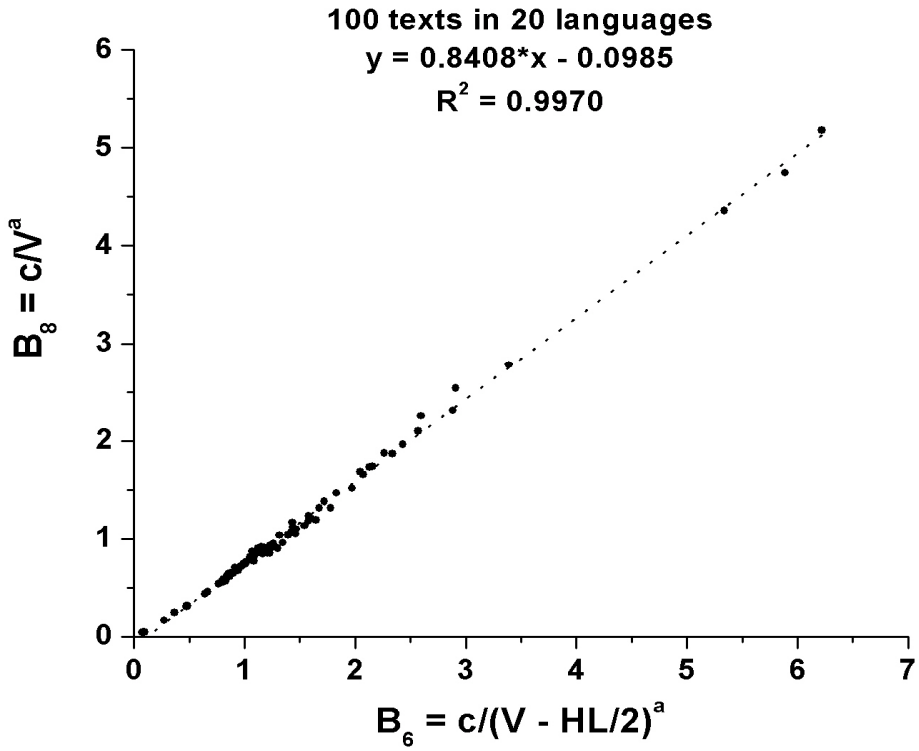


Figure 7.2. The relationship between indicators B_6 and B_8

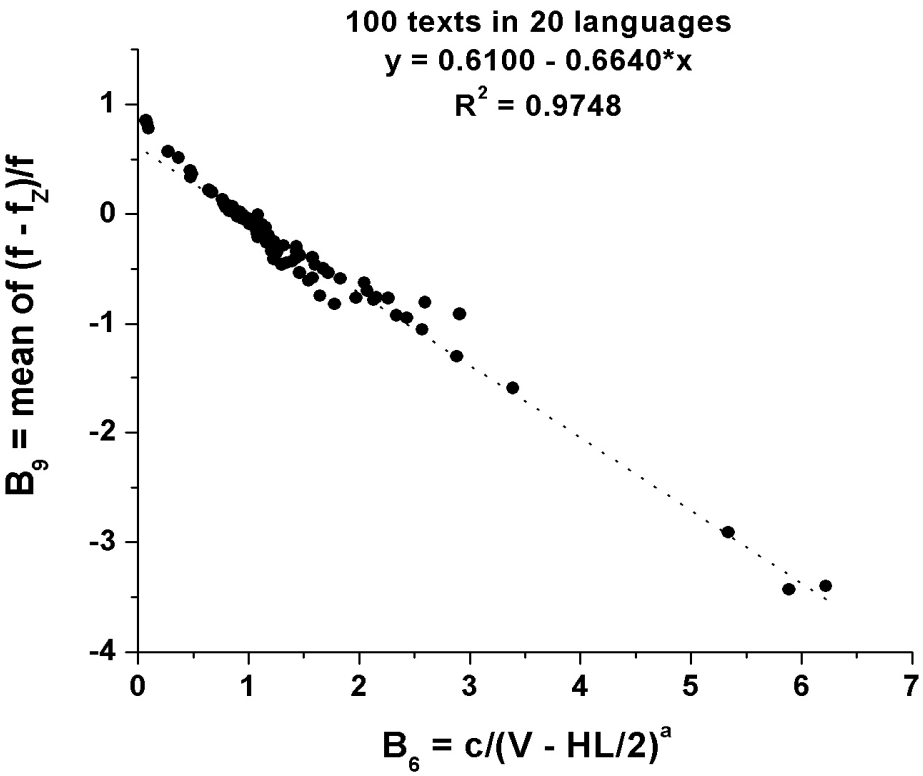


Figure 7.3. The relationship between indicators B_6 and B_9

Finally, as seen in Figure 7.3, B_9 decreases linearly in terms of B_6 according to

(7.7) $B_9 = -0.6640B_6 + 0.6100$

Even if one replaces the Zipf function by another function, its behaviour is still appropriate for language typology.

The last indicator scrutinized here is based only on V and $f(1)$ which, as has been shown, develop differently in dependence on the morphological character of the language. At the same time, they are different for every text but their ratio may display some kind of constancy. We define the simple indicator

(7.8) $$B_{10} = \frac{V-1}{f(1)-1}$$

which takes values in the interval [0, ∞]. This is not a proportion, i.e. it cannot be processed statistically in that form, but in the ratio of the moduli of two vectors. The lower boundary can be attained only asymptotically, the upper one only in extreme texts which are not interesting for linguistics. In "normal" texts, the index attains special values which will be examined here. In order to get some more general results, we transform the index in radians and obtain

(7.9) $\theta = arctan(B_{10})$ *radians*.

The geometrical interpretation of this relationship is straightforward and is illustrated in Figure 7.4.

For example in Goethe´s "Erlkönig" there are $V = 124$ and $f(1) = 11$, hence from Eq. (7.8) we obtain $B_{10} = 123/10 = 12.3$. Transforming it in radians with the help of Eq. (7.9), we obtain $\theta = \arctan(12.3) = 1.4897$ radians. Obviously, the lower the maximal word frequency $f(1)$ and the greater the vocabulary V, the greater is the angle θ and at the lowest limit $f(1) = 1$ we have the maximum theoretical values $I = \infty$ and $\theta = \pi / 2 = 1.507...$

Now several questions arise:

(1) What is the actual interval of the transformed indicator (7.9), i.e. what are the effective limits of the indicator?

(2) How does the indicator change in the course of the text? Does it display chaotic or regular oscillation or does it have a "smooth" course? Is it a constant or a trend?

(3) Considering the values of θ for different texts, can we distinguish authors, genres or even languages, i.e. do special text types display special behaviour which can be exploited for typological conclusions?

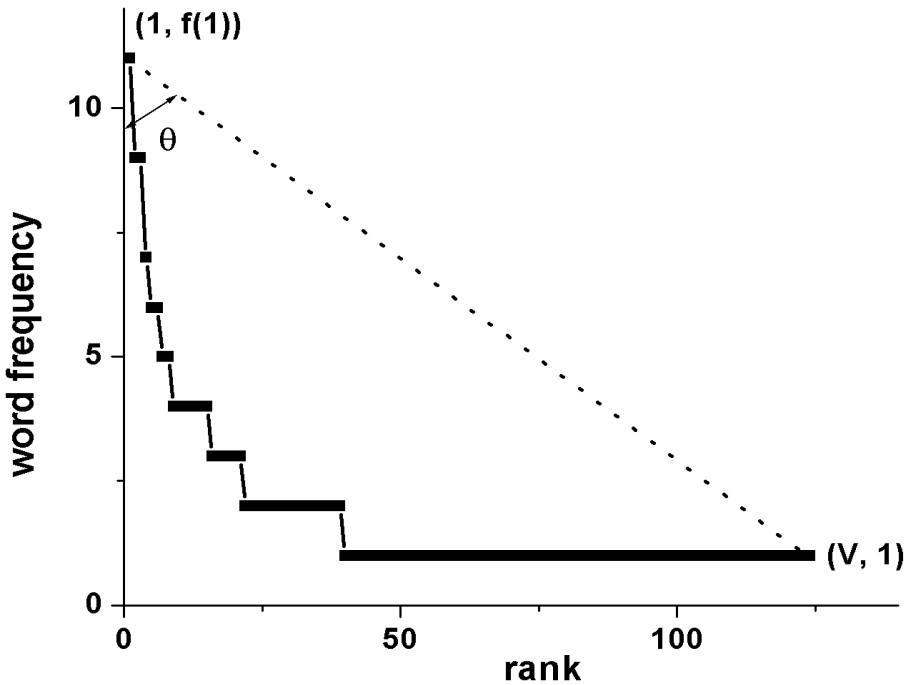


Figure 7.4. Definition of the θ indicator
(plotted data correspond to Goethe´s "Erlkönig")

Hence the index has different aspects: static, dynamic, within-language (genres, style) and between-language (typological, morphological) one. In some cases the frame of reference is the text length measured in terms of word(-form) numbers; with the dynamic aspect the frames can be words themselves (N), verses, sentences, chapters etc. Since no investigation in this direction has been made up to now, we shall try several possibilities.

We shall scrutinize these questions using texts in 20 languages taken from Popescu et al. (2009). Let us present some data in which the reference number is text length N (number of word forms in text). The indicator θ is a simple characteristic of individual texts (see Table 7.3).

We see that empirically the indicator θ lies in the interval $\theta \in$ <0.8565, 1.5461>. Though other texts in other languages may display more extreme behaviour, we know for sure that the theoretical upper limit is $\theta = \pi/2 = 1.5708$ and may preliminarily tentatively conjecture that the lower limit tends to half of the golden section $(1+\sqrt{5})/2 = 1.618$, namely to 0.809. The appearance of the golden section in this context is not surprising since it was discovered in other context of word frequencies, too (cf. Popescu, Altmann 2007; Tuzzi, Popescu,

Altmann 2009). The placing of θ in 176 texts in 20 languages is shown in Figure 7.5.

Table 7.3
The indicator $\theta = \arctan[(V - 1)/(f(1) - 1)]$ radians for 100 texts in 20 languages

ID	*N*	*V*	*f*(1)	θ radians	ID	*N*	*V*	*f*(1)	θ radians
B 01	761	400	40	1.4734	Lk 03	809	272	62	1.3494
B 02	352	201	13	1.5109	Lk 04	219	116	18	1.4240
B 03	515	285	15	1.5215	Lt 01	3311	2211	133	1.5111
B 04	483	286	21	1.5007	Lt 02	4010	2334	190	1.4900
B 05	406	238	19	1.4950	Lt 03	4931	2703	103	1.5331
Cz 01	1044	638	58	1.4816	Lt 04	4285	1910	99	1.5195
Cz 02	984	543	56	1.4697	Lt 05	1354	909	33	1.5356
Cz 03	2858	1274	182	1.4296	Lt 06	829	609	19	1.5412
Cz 04	522	323	27	1.4902	M 01	2062	398	152	1.2073
Cz 05	999	556	84	1.4223	M 02	1175	277	127	1.1425
E 01	2330	939	126	1.4383	M 03	1434	277	128	1.1395
E 02	2971	1017	168	1.4079	M 04	1289	326	137	1.1745
E 03	3247	1001	229	1.3466	M 05	3620	514	234	1.1445
E 04	4622	1232	366	1.2825	Mq 01	2330	289	247	0.8639
E 05	4760	1495	297	1.3752	Mq 02	457	150	42	1.3023
E 07	5004	1597	237	1.4240	Mq 03	1509	301	218	0.9446
E 13	11265	1659	780	1.1316	Mr 001	2998	1555	75	1.5232
G 05	559	332	30	1.4834	Mr 018	4062	1788	126	1.5010
G 09	653	379	30	1.4942	Mr 026	4146	2038	84	1.5301
G 10	480	301	18	1.5142	Mr 027	4128	1400	92	1.5058
G 11	468	297	18	1.5134	Mr 288	4060	2079	84	1.5309
G 12	251	169	14	1.4936	R 01	1738	843	62	1.4985
G 14	184	129	10	1.5006	R 02	2279	1179	110	1.4785
G 17	225	124	11	1.4897	R 03	1264	719	65	1.4819
H 01	2044	1079	225	1.3659	R 04	1284	729	49	1.5050
H 02	1288	789	130	1.4085	R 05	1032	567	46	1.4915
H 03	403	291	48	1.4101	R 06	695	432	30	1.5036
H 04	936	609	76	1.4481	Rt 01	968	223	111	1.1108
H 05	413	290	32	1.4639	Rt 02	845	214	69	1.2618
Hw 03	3507	521	277	1.0828	Rt 03	892	207	66	1.2651
Hw 04	7892	744	535	0.9476	Rt 04	625	181	49	1.3102
Hw 05	7620	680	416	1.0222	Rt 05	1059	197	74	1.2143

Hw 06	12356	1039	901	0.8565	Ru 01	753	422	31	1.4997
I 01	11760	3667	388	1.4656	Ru 02	2595	1240	138	1.4607
I 02	6064	2203	257	1.4551	Ru 03	3853	1792	144	1.4911
I 03	854	483	64	1.4408	Ru 04	6025	2536	228	1.4815
I 04	3258	1237	118	1.4764	Ru 05	17205	6073	701	1.4560
I 05	1129	512	42	1.4907	Sl 01	756	457	47	1.4703
In 01	376	221	16	1.5027	Sl 02	1371	603	66	1.4632
In 02	373	209	18	1.4892	Sl 03	1966	907	102	1.4598
In 03	347	194	14	1.5035	Sl 04	3491	1102	328	1.2821
In 04	343	213	11	1.5237	Sl 05	5588	2223	193	1.4846
In 05	414	188	16	1.4908	Sm 01	1487	267	159	1.0348
Kn 003	3188	1833	74	1.531	Sm 02	1171	222	103	1.1384
Kn 004	1050	720	23	1.5402	Sm 03	617	140	45	1.2642
Kn 005	4869	2477	101	1.5304	Sm 04	736	153	78	1.1019
Kn 006	5231	2433	74	1.5408	Sm 05	447	124	39	1.2712
Kn 011	4541	2516	63	1.5461	T 01	1551	611	89	1.4275
Lk 01	345	174	20	1.4614	T 02	1827	720	107	1.4244
Lk 02	1633	479	124	1.3189	T 03	2054	645	128	1.3761

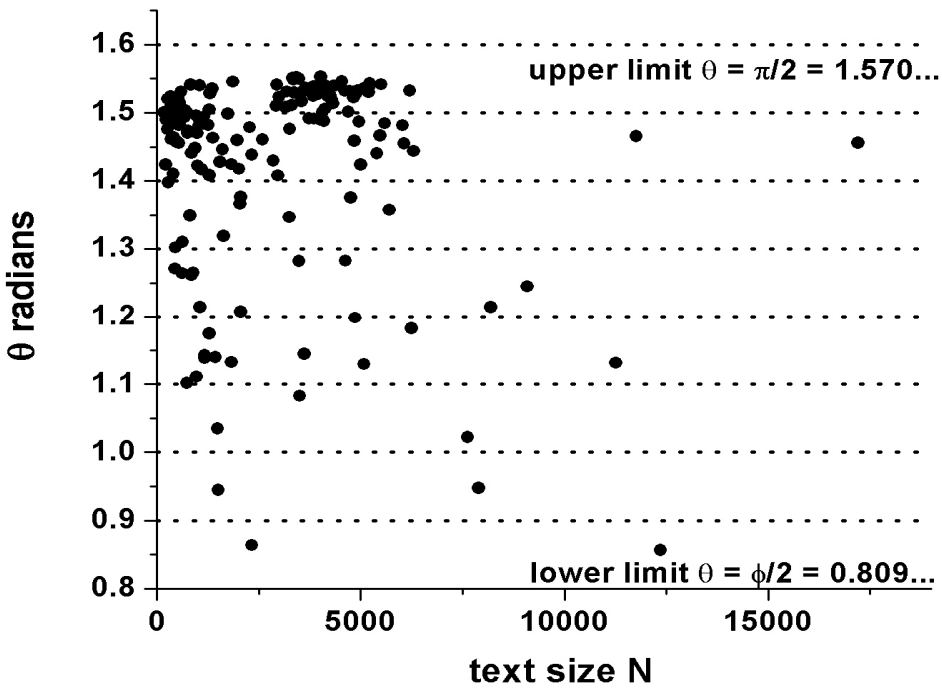


Figure 7.5. Indicator θ of 176 texts in 20 languages in terms of text size

Since the lowest value of the index is displayed in Marquesan and the highest in Kannada, we may conjecture that even if this result does not exactly correlate with the morphological status of the languages; nevertheless, there may exist differences between languages. Let us first present a table of mean θ values for all texts in individual languages in Table 7.4. As can be seen, there is a trend from synthetism to analytism but the number of texts is preliminarily too small and some of them are too short to give a reliable result. Nevertheless, further research can improve this result.

Table 7.4
Mean analytism θ indicator of 20 languages

Language	mean θ radians	number of texts
Kannada	1.5377	5
Latin	1.5217	6
Marathi	1.5182	5
Indonesian	1.5020	5
Bulgarian	1.5003	5
German	1.4984	7
Romanian	1.4932	6
Rusian	1.4778	5
Italian	1.4657	5
Czech	1.4587	5
Slovenian	1.4320	5
Hungarian	1.4193	5
Tagalog	1.4093	3
Lakota	1.3884	4
English	1.3437	7
Rarotongan	1.2324	5
Samoan	1.1621	5
Maori	1.1617	5
Marquesan	1.0369	3
Hawaiian	0.9773	5

At least the fact that there is a difference between languages can be corroborated in more detail by a graphical presentation of more texts in selected languages, as shown in Figure 7.6 for Kannada, Marathi, German, Hawaiian, and Marquesan. The source of Kannada and Marathi texts is indicated in the book "Word frequency studies" (Popescu et al. 2009), that of German texts is given in the Appendix to the present book, and the additional Polynesian texts have been collected on Internet from "The Hawaiian Romance of Laieikawai, by Anonymous" at http://www.gutenberg.org/files/13603/ 13603.txt for Hawaiian, respec-

tively at http://www.lds.org/conference/talk/display/0,5232,23-14-659-31,00.html and at http://www.fides.org/ita/approfondire/preghiere/prayer_031106_tahiti.doc for Marquesan.

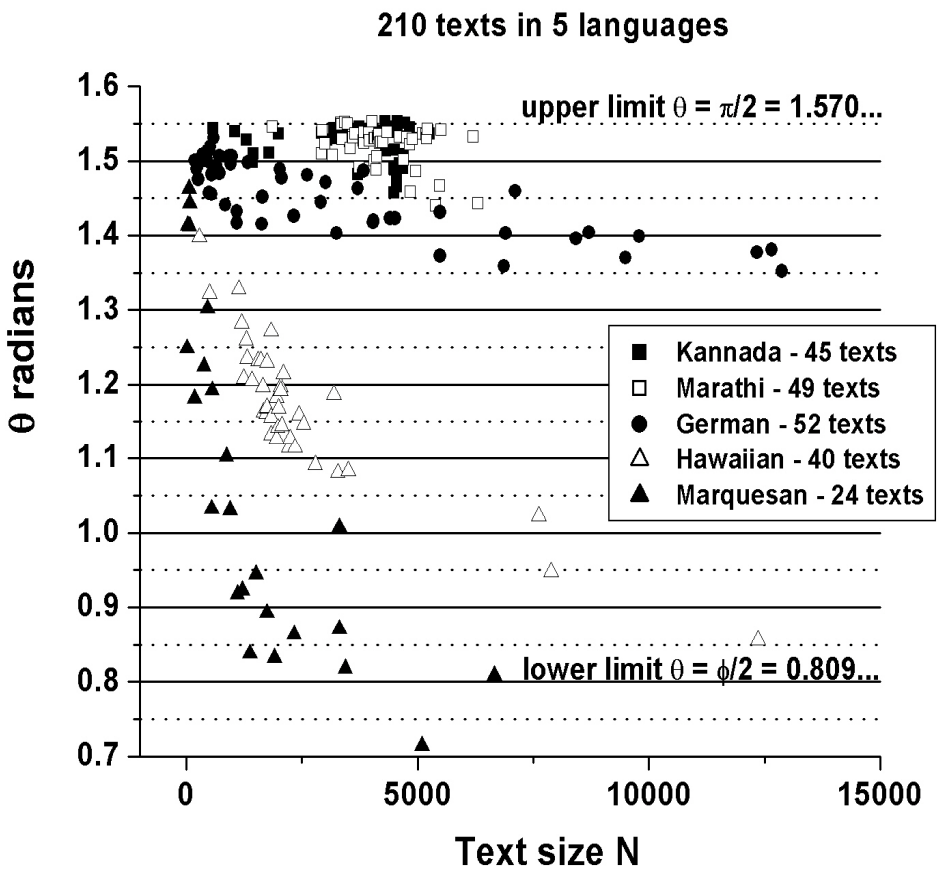


Figure 7.6. The θ indicator differentiated according to languages

It can be seen that in every language θ has its own course depending on text length N. The higher the degree of synthetism in language, the longer θ remains on a high level. The highest average θ level belongs to the most synthetic Kannada (1.525 radians) and Marathi (1.522 radians). By contrast, in analytic languages θ decreases very quickly and evidently it converges to a certain limit $\theta_{\min}$. For the sake of lucidity we present these languages separately in Table 7.5 and in Figures 7.7 to 7.9.

Table 7.5
The θ indicator for five selected languages:
Kannada, Marathi, German, Hawaiian, and Marquesan

ID	N	V	$f(1)$	B_{10}	θ radians
G 01	1095	530	83	6.4512	1.4170
G 02	845	361	48	7.6596	1.4410
G 03	500	281	33	8.7500	1.4570
G 04	545	269	32	8.6452	1.4556
G 05	559	332	30	11.4138	1.4834
G 06	545	326	30	11.2069	1.4818
G 07	263	169	17	10.5000	1.4758
G 08	965	509	39	13.3684	1.4961
G 09	653	379	30	13.0345	1.4942
G 10	480	301	18	17.6471	1.5142
G 11	468	297	18	17.4118	1.5134
G 12	251	169	14	12.9231	1.4936
G 13	460	253	19	14.0000	1.4995
G 14	184	129	10	14.2222	1.5006
G 15	593	378	16	25.1333	1.5310
G 16	518	292	16	19.4000	1.5193
G 17	225	124	11	12.3000	1.4897
G 18	356	227	15	16.1429	1.5089
G 19	986	561	37	15.5556	1.5066
G 20	683	411	35	12.0588	1.4881
G 21	715	421	28	15.5556	1.5066
G 22	929	502	33	15.6563	1.5070
G 23	1328	718	53	13.7885	1.4984
G 24	717	449	40	11.4872	1.4840
G 25	2025	1024	85	12.1786	1.4889
G 26	2063	1029	97	10.7083	1.4777
G 27	4047	963	147	6.5890	1.4202
G 28	2326	681	100	6.8687	1.4262
G 29	1630	512	81	6.3875	1.4155
G 30	1096	374	53	7.1731	1.4323
G 31	4412	1052	157	6.7372	1.4234
G 32	1649	570	69	8.3676	1.4519
G 33	4515	1051	157	6.7308	1.4233
G 34	2909	1036	132	7.9008	1.4449

G 35	3253	841	143	5.9155	1.4033
G 36	5490	1343	270	4.9888	1.3730
G 37	6869	1463	315	4.6561	1.3592
G 38	4043	1148	178	6.4802	1.4177
G 39	3834	1483	126	11.8560	1.4867
G 40	2617	1035	94	11.1183	1.4811
G 41	3709	1354	147	9.2671	1.4633
G 42	3012	1264	127	10.0238	1.4714
G 43	7110	2469	276	8.9745	1.4598
G 44	5486	1824	257	7.1211	1.4313
G 45	9788	2614	454	5.7682	1.3991
G 46	12656	3073	591	5.2068	1.3810
G 47	6901	1939	329	5.9085	1.4031
G 48	9493	2385	485	4.9256	1.3705
G 49	12879	2951	656	4.5038	1.3523
G 50	8426	2276	403	5.6592	1.3959
G 51	8704	2413	406	5.9556	1.4044
G 52	12335	3042	596	5.1109	1.3776
Hw 01	282	104	19	5.7222	1.3978
Hw 02	1829	257	121	2.1333	1.1325
Hw 03	3507	521	277	1.8841	1.0828
Hw 04	7892	744	535	1.3914	0.9476
Hw 05	7620	680	416	1.6361	1.0222
Hw 06	12356	1039	901	1.1533	0.8565
Hw 07	506	135	35	3.9412	1.3223
Hw 08	2348	321	158	2.0382	1.1147
Hw 09	2788	322	168	1.9222	1.0911
Hw 10	2537	349	158	2.2166	1.1470
Hw 11	3191	387	157	2.4744	1.1867
Hw 12	1832	309	96	3.2421	1.2716
Hw 13	1996	323	138	2.3504	1.1685
Hw 14	1296	219	71	3.1143	1.2601
Hw 15	1658	266	105	2.5481	1.1968
Hw 16	3273	357	191	1.8737	1.0805
Hw 17	1547	234	83	2.8415	1.2324
Hw 18	1743	252	90	2.8202	1.2300
Hw 19	1617	268	95	2.8404	1.2323
Hw 20	2105	302	113	2.6875	1.2146

Hw 21	1132	219	55	4.0370	1.3280
Hw 22	1199	227	68	3.3731	1.2826
Hw 23	1314	233	82	2.8642	1.2349
Hw 24	2046	302	119	2.5508	1.1972
Hw 25	1840	260	115	2.2719	1.1562
Hw 26	1951	280	115	2.4474	1.1829
Hw 27	1666	244	106	2.3143	1.1629
Hw 28	2245	306	146	2.1034	1.1270
Hw 29	2223	299	147	2.0411	1.1152
Hw 30	1731	245	107	2.3019	1.1610
Hw 31	1976	292	134	2.1880	1.1421
Hw 32	1953	300	143	2.1056	1.1274
Hw 33	1738	269	115	2.3509	1.1686
Hw 34	2435	365	160	2.2893	1.1590
Hw 35	1245	228	87	2.6395	1.2087
Hw 36	1423	291	112	2.6126	1.2052
Hw 37	2046	305	122	2.5124	1.1920
Hw 38	1740	260	111	2.3545	1.1692
Hw 39	2100	315	118	2.6838	1.2141
Hw 40	2063	285	130	2.2016	1.1444
Kn 001	3713	1664	149	11.2365	1.4820
Kn 002	4508	1738	98	17.9072	1.5150
Kn 003	3188	1833	74	25.0959	1.5310
Kn 004	1050	720	23	32.6818	1.5402
Kn 005	4869	2477	101	24.7600	1.5304
Kn 006	5231	2433	74	33.3151	1.5408
Kn 007	4434	2724	88	31.2989	1.5389
Kn 008	4393	2781	78	36.1039	1.5431
Kn 009	3733	2222	57	39.6607	1.5456
Kn 010	4828	2613	105	25.1154	1.5310
Kn 011	4541	2516	63	40.5645	1.5461
Kn 012	4141	1842	58	32.2982	1.5398
Kn 013	1302	807	35	23.7059	1.5286
Kn 015	4456	2360	59	40.6724	1.5462
Kn 016	4735	2356	93	25.5978	1.5318
Kn 017	4316	2122	122	17.5289	1.5138
Kn 018	4483	1782	147	12.1986	1.4890
Kn 019	1787	833	51	16.6400	1.5108
Kn 020	4556	1755	161	10.9625	1.4798

Kn 021	1455	790	49	16.4375	1.5100
Kn 022	4554	1764	186	9.5297	1.4662
Kn 023	4685	1738	140	12.4964	1.4909
Kn 024	4588	2896	58	50.7895	1.5511
Kn 025	4559	2635	48	56.0426	1.5530
Kn 026	3716	2072	60	35.1017	1.5423
Kn 030	4499	2005	167	12.0723	1.4882
Kn 031	4692	1920	119	16.2627	1.5094
Kn 044	2000	1281	45	29.0909	1.5364
Kn 045	4304	2747	49	57.2083	1.5533
Kn 046	4723	3077	63	49.6129	1.5506
Kn 047	4084	2726	90	30.6180	1.5381
Kn 068	3530	2026	91	22.5000	1.5264
Kn 069	4567	2571	79	32.9487	1.5405
Kn 070	3184	1874	51	37.4600	1.5441
Kn 071	5258	2811	97	29.2708	1.5366
Kn 075	4485	1635	187	8.7849	1.4575
Kn 079	4610	1794	135	13.3806	1.4962
Kn 080	4829	2579	68	38.4776	1.5448
Kn 081	3247	1830	60	31.0000	1.5385
Kn 082	1435	873	64	13.8413	1.4987
Kn 101	2930	1720	58	30.1579	1.5376
Kn 102	3801	2348	72	33.0563	1.5406
Kn 104	578	412	12	37.3636	1.5440
Kn 105	3043	1577	67	23.8788	1.5289
Kn 186	3382	1806	65	28.2031	1.5354
Mq 01	2330	289	247	1.1707	0.8639
Mq 02	457	150	42	3.6341	1.3023
Mq 03	1509	301	218	1.3825	0.9446
Mq 04	14	10	4	3.0000	1.2490
Mq 05	76	40	6	7.8000	1.4433
Mq 06	67	38	5	9.2500	1.4631
Mq 07	26	20	4	6.3333	1.4142
Mq 08	178	79	33	2.4375	1.1815
Mq 09	69	45	8	6.2857	1.4130
Mq 10	380	137	50	2.7755	1.2250
Mq 11	1746	320	258	1.2412	0.8926
Mq 12	1376	220	198	1.1117	0.8382
Mq 13	550	155	93	1.6739	1.0323
Mq 14	950	223	134	1.6692	1.0310

Mq 15	1213	235	178	1.3220	0.9232
Mq 16	559	152	61	2.5167	1.1926
Mq 17	868	191	97	1.9792	1.1029
Mq 18	1907	302	275	1.0985	0.8323
Mq 19	1105	193	148	1.3061	0.9174
Mq 20	6659	519	495	1.0486	0.8091
Mq 21	3315	330	278	1.1877	0.8710
Mq 22	3306	362	229	1.5833	1.0075
Mq 23	5102	434	500	0.8677	0.7147
Mq 24	3435	461	432	1.0673	0.8179
Mr 001	2998	1555	75	21.0000	1.5232
Mr 002	2922	1186	73	16.4583	1.5101
Mr 003	4140	1731	68	25.8209	1.5321
Mr 004	6304	2451	314	7.8275	1.4437
Mr 005	4957	2029	172	11.8596	1.4867
Mr 006	3735	1503	120	12.6218	1.4917
Mr 007	3162	1262	80	15.9620	1.5082
Mr 008	5477	1807	190	9.5556	1.4665
Mr 009	6206	2387	93	25.9348	1.5323
Mr 010	5394	1650	217	7.6343	1.4405
Mr 015	4693	1947	136	14.4148	1.5015
Mr 016	3642	1831	63	29.5161	1.5369
Mr 017	4170	1853	67	28.0606	1.5352
Mr 018	4062	1788	126	14.2960	1.5010
Mr 020	3943	1825	62	29.9016	1.5374
Mr 021	3846	1793	58	31.4386	1.5390
Mr 022	4099	1703	142	12.0709	1.4881
Mr 023	4142	1872	72	26.3521	1.5329
Mr 024	4255	1731	80	21.8987	1.5252
Mr 026	4146	2038	84	24.5422	1.5301
Mr 027	4128	1400	92	15.3736	1.5058
Mr 028	5191	2386	86	28.0588	1.5352
Mr 029	3424	1412	28	52.2593	1.5517
Mr 030	5504	2911	86	34.2353	1.5416
Mr 031	5105	2617	91	29.0667	1.5364
Mr 032	5195	2382	98	24.5464	1.5301
Mr 033	4339	2217	71	31.6571	1.5392
Mr 034	3489	1865	40	47.7949	1.5499
Mr 035	1862	1115	29	39.7857	1.5457
Mr 036	4205	2070	96	21.7789	1.5249

Mr 038	4078	1607	66	24.7077	1.5303
Mr 040	5218	2877	81	35.9500	1.5430
Mr 043	3356	1962	44	45.6047	1.5489
Mr 046	4186	1458	68	21.7463	1.5248
Mr 052	3549	1628	89	18.4886	1.5168
Mr 149	2946	1547	47	33.6087	1.5411
Mr 150	3372	1523	64	24.1587	1.5294
Mr 151	4843	1702	192	8.9058	1.4590
Mr 154	3601	1719	68	25.6418	1.5318
Mr 288	4060	2079	84	25.0361	1.5309
Mr 289	4831	2312	112	20.8198	1.5228
Mr 290	4025	2319	42	56.5366	1.5531
Mr 291	3954	1957	86	23.0118	1.5274
Mr 292	4765	2197	88	25.2414	1.5312
Mr 293	3337	2006	41	50.1250	1.5508
Mr 294	3825	1931	85	22.9762	1.5273
Mr 295	4895	2322	97	24.1771	1.5295
Mr 296	3836	1970	92	21.6374	1.5246
Mr 297	4605	2278	88	26.1724	1.5326

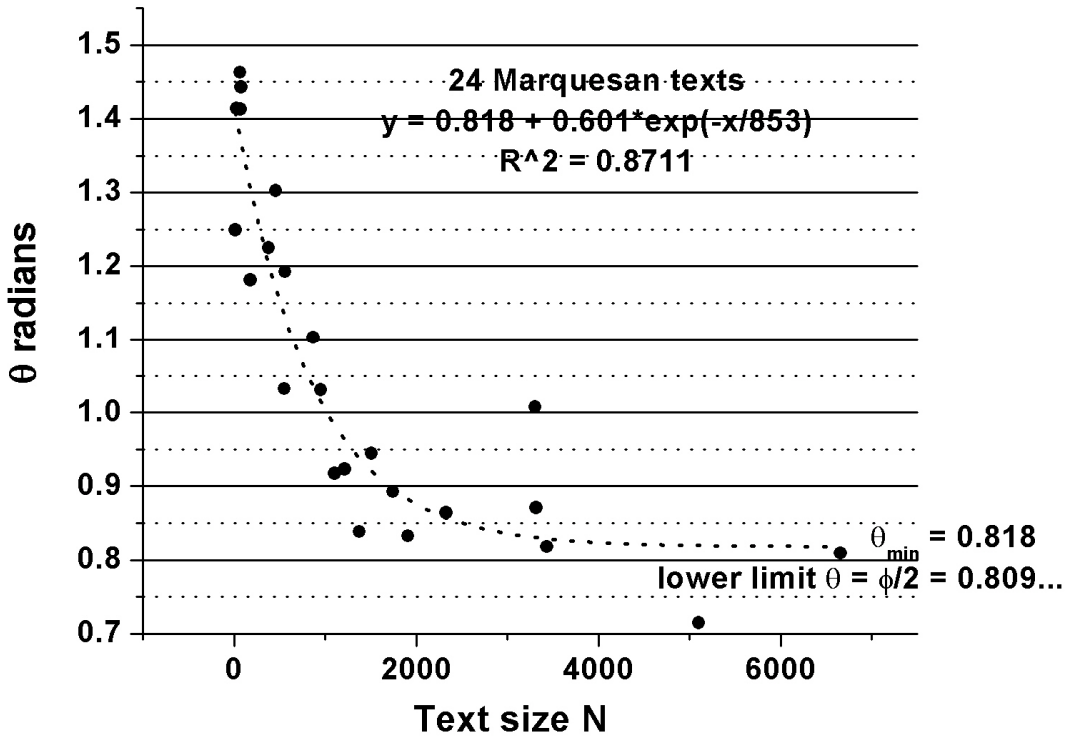


Figure 7.7. The course of θ in 24 Marquesan texts in terms of text size

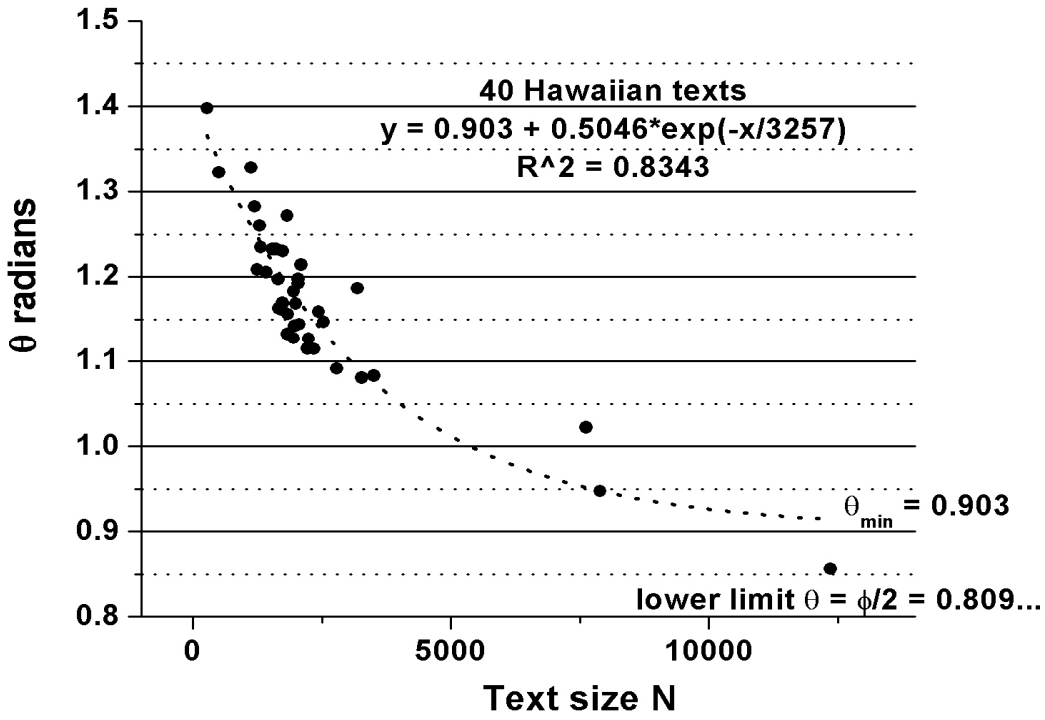


Figure 7.8. The course of θ in 40 Hawaiian texts in terms of text size

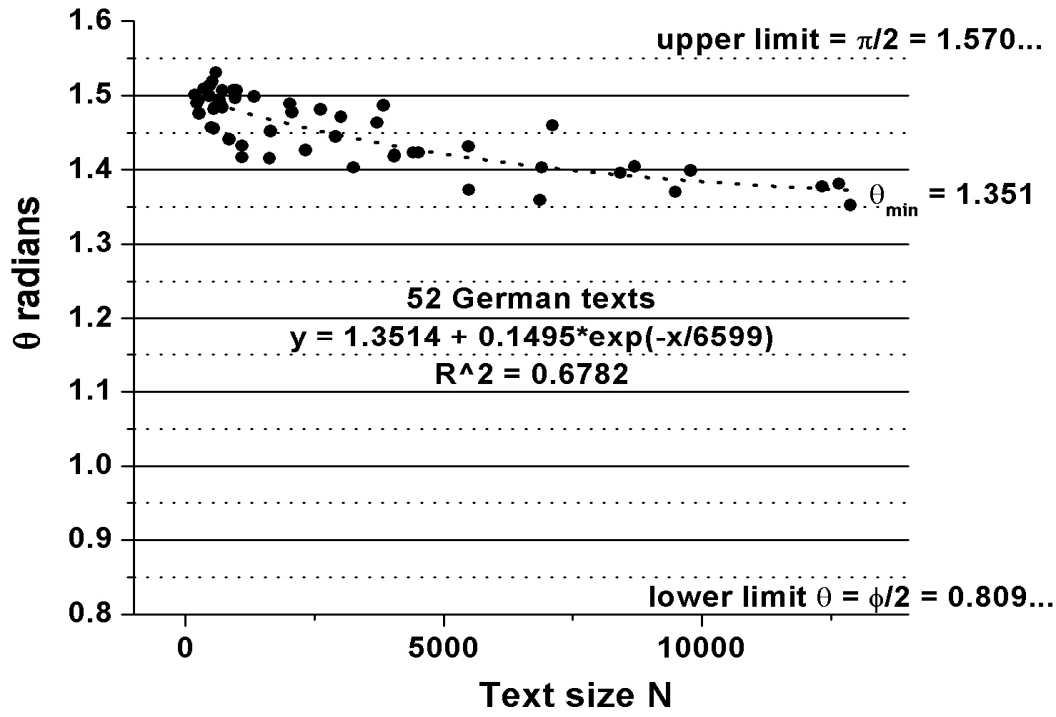


Figure 7.9. The course of θ in 52 German texts in terms of text size

The decrease seems to be very regular and can be preliminarily captured by the curve

(7.10) $\theta = \theta_{\min} + A\exp(-N/K)$

which is an empirical solution changing with every analyzed language. In any case it shows the decreasing exponential course of θ with increasing text size, tending to different limits $\theta_{\min}$ for different languages, such as to 0.818 radians for Marquesan, to 0.903 radians for Hawaiian, and to 1.351 radians for German. In Marquesan (Fig. 7.7) we see two outliers one of which is positioned below the limit stated by the golden section. In Hawaiian (Fig. 7.8) and in German (Fig. 7.9) the course is relatively smooth.

A very simple indicator which can be used both for text typology as well as (cum grano salis) language typology is the intuitive shape factor

(7.11) $$S = \frac{L}{h-1},$$

i.e. the ratio of arc length (as the major dimension) to the *h*-point range (as the minor dimension). Its different aspects and uses can be found in many places on the Internet (cf. http://en.wikipedia.org/wiki/Compactness_measure_of_a_shape or http://en.wikipedia.org/wiki/Shape_factor). It concerns merely the shape elements of the rank-frequency sequence and can have very variegated outcomes.

Firstly let us illustrate the results using 100 texts in 20 languages as can be seen in Table 7.6

Table 7.6
The shape ratio *S* for 100 texts in 20 languages

ID	*N*	*V*	*f*(1)	*h*	*L*	*S* = *L*/(*h*-1)
B 01	761	400	40	10	428	47.61
B 02	352	201	13	8	205	29.34
B 03	515	285	15	9	290	36.23
B 04	483	286	21	8	297	42.43
B 05	406	238	19	7	247	41.22
Cz 01	1044	638	58	9	684	85.52
Cz 02	984	543	56	11	586	58.62
Cz 03	2858	1274	182	19	1432	79.56
Cz 04	522	323	27	7	342	57.00
Cz 05	999	556	84	9	627	78.37
E 01	2330	939	126	16	1043	69.52
E 02	2971	1017	168	22	1157	55.11

E 03	3247	1001	229	19	1205	66.94
E 04	4622	1232	366	23	1567	71.24
E 05	4760	1495	297	26	1761	70.43
E 07	5004	1597	237	25	1801	75.03
E 13	11265	1659	780	41	2388	59.71
G 05	559	332	30	8	351	50.20
G 09	653	379	30	9	389	48.68
G 10	480	301	18	7	310	51.64
G 11	468	297	18	7	307	51.13
G 12	251	169	14	6	175	35.09
G 14	184	129	10	5	133	33.14
G 17	225	124	11	6	128	25.59
H 01	2044	1079	225	12	1289	117.17
H 02	1288	789	130	8	907	129.60
H 03	403	291	48	4	332	110.81
H 04	936	609	76	7	674	112.34
H 05	413	290	32	6	314	62.88
Hw 03	3507	521	277	26	764	30.57
Hw 04	7892	744	535	38	1229	33.22
Hw 05	7620	680	416	38	1047	28.31
Hw 06	12356	1039	901	44	1877	43.64
I 01	11760	3667	388	37	4007	111.31
I 02	6064	2203	257	25	2426	101.10
I 03	854	483	64	10	534	59.37
I 04	3258	1237	118	21	1330	66.48
I 05	1129	512	42	12	537	48.86
In 01	376	221	16	6	228	45.70
In 02	373	209	18	7	219	36.44
In 03	347	194	14	6	200	39.97
In 04	343	213	11	5	217	54.34
In 05	414	188	16	8	196	27.95
Kn 003	3188	1833	74	13	1891	157.59
Kn 004	1050	720	23	7	733	122.21
Kn 005	4869	2477	101	16	2558	170.56
Kn 006	5231	2433	74	20	2481	130.60
Kn 011	4541	2516	63	17	2558	159.86
Lk 01	345	174	20	8	185	26.40
Lk 02	1633	479	124	17	580	36.25

Lk 03	809	272	62	12	318	28.88
Lk 04	219	116	18	6	126	25.11
Lt 01	3311	2211	133	12	2328	211.64
Lt 02	4010	2334	190	18	2502	147.18
Lt 03	4931	2703	103	19	2783	154.61
Lt 04	4285	1910	99	20	1983	104.37
Lt 05	1354	909	33	8	930	132.86
Lt 06	829	609	19	7	621	103.50
M 01	2062	398	152	18	527	31.00
M 02	1175	277	127	15	386	27.57
M 03	1434	277	128	17	385	24.04
M 04	1289	326	137	15	444	31.74
M 05	3620	514	234	26	715	28.61
Mq 01	2330	289	247	22	507	24.14
Mq 02	457	150	42	10	179	19.84
Mq 03	1509	301	218	14	500	38.49
Mr 001	2998	1555	75	14	1612	124.03
Mr 018	4062	1788	126	20	1890	99.49
Mr 026	4146	2038	84	19	2099	116.61
Mr 027	4128	1400	92	21	1468	73.38
Mr 288	4060	2079	84	17	2141	133.81
R 01	1738	843	62	14	886	68.18
R 02	2279	1179	110	16	1269	84.60
R 03	1264	719	65	12	770	70.02
R 04	1284	729	49	10	764	84.93
R 05	1032	567	46	11	599	59.92
R 06	695	432	30	10	452	50.19
Rt 01	968	223	111	14	316	24.30
Rt 02	845	214	69	13	265	22.06
Rt 03	892	207	66	13	256	21.32
Rt 04	625	181	49	11	216	21.56
Rt 05	1059	197	74	15	251	17.91
Ru 01	753	422	31	8	441	63.01
Ru 02	2595	1240	138	16	1357	90.45
Ru 03	3853	1792	144	21	1909	95.45
Ru 04	6025	2536	228	25	2732	113.82
Ru 05	17205	6073	701	41	6722	168.05
Sl 01	756	457	47	9	494	61.72

Sl 02	1371	603	66	13	651	54.26
Sl 03	1966	907	102	13	991	82.58
Sl 04	3491	1102	328	21	1404	70.21
Sl 05	5588	2223	193	25	2385	99.39
Sm 01	1487	267	159	17	403	25.20
Sm 02	1171	222	103	15	304	21.71
Sm 03	617	140	45	13	168	14.03
Sm 04	736	153	78	12	214	19.47
Sm 05	447	124	39	11	149	14.95
T 01	1551	611	89	14	681	52.38
T 02	1827	720	107	15	807	57.68
T 03	2054	645	128	19	749	41.58

The typological capabilities of the shape ratio are evidenced in Table 7.7, showing the ranking of languages by decreasing mean *S* values, in the right general order, from synthetical to analytical languages.

Table 7.7
Mean analytism *S* indicator of 20 languages

Language	mean *S*	number of texts
Kannada	148.16	5
Latin	142.36	6
Marathi	109.47	5
Hungarian	106.56	5
Russian	106.16	5
Italian	77.42	5
Slovenian	73.63	5
Czech	71.81	5
Romanian	69.64	6
English	66.86	7
Tagalog	50.55	3
German	42.21	7
Indonesian	40.88	5
Bulgarian	39.36	5
Hawaiian	33.94	4
Lakota	29.16	4
Maori	28.59	5
Marquesan	27.49	3
Rarotongan	21.43	5
Samoan	19.07	5

As an application to a single language and with better statistics, Table 7.8 presents the shape ratio of 253 texts of 26 German writers and Table 7.9 the corresponding mean value for each author.

Table 7.8
The shape ratio of 253 texts of 26 German writers

ID	*N*	*V*	*f*(1)	*h*	*L*	*S* = *L*/(*h* - 1)
Arnim 01	7846	2221	271	33	2448	76.50
Arnim 02	1201	564	46	13	595	49.54
Arnim 03	4167	1429	189	26	1588	63.52
Busch 01	15820	4642	527	44	5112	118.88
Chamisso 01	2210	884	82	18	944	55.53
Chamisso 02	1847	808	84	16	872	58.13
Chamisso 03	1428	630	70	14	684	52.62
Chamisso 04	3205	1209	123	20	1305	68.68
Chamisso 05	2108	853	79	18	911	53.59
Chamisso 06	1948	801	75	17	853	53.31
Chamisso 07	1362	670	44	13	698	58.17
Chamisso 08	1870	788	80	16	848	56.53
Chamisso 09	1320	593	96	14	673	51.77
Chamisso 10	1012	536	52	11	575	57.50
Chamisso 11	1386	656	66	14	705	54.23
Droste 01	16172	4064	525	49	4528	94.33
Droste 02	884	492	48	10	527	61.14
Droste 03	700	425	31	9	444	55.51
Droste 04	786	408	34	11	430	45.23
Droste 05	1274	657	51	13	692	60.18
Droste 08	965	509	39	11	535	53.46
Eichendorff 01	3080	1079	177	21	1228	61.40
Eichendorff 02	4100	1287	210	25	1466	61.08
Eichendorff 03	4342	1334	182	28	1482	54.89
Eichendorff 04	1781	739	79	16	799	53.27
Eichendorff 05	1680	699	70	16	750	50.00
Eichendorff 06	3223	1059	130	22	1163	55.38
Eichendorff 07	2594	932	121	20	1031	54.26
Eichendorff 08	3987	1320	159	25	1447	60.29
Eichendorff 09	3285	1185	155	22	1315	62.62
Eichendorff 10	3052	1073	131	22	1178	56.10

Goethe 01	7554	2222	318	33	2502	78.19
Goethe 05	559	332	30	8	351	50.14
Goethe 09	653	379	30	9	398	49.75
Goethe 10	480	301	18	7	310	51.67
Goethe 11	468	297	18	7	307	51.17
Goethe 12	251	169	14	6	175	35.00
Goethe 14	184	129	10	5	133	33.25
Goethe 17	225	124	11	6	128	25.60
Heine 01	19522	5769	939	47	6648	146.11
Heine 02	603	361	50	9	400	53.36
Heine 03	394	211	21	7	222	37.03
Heine 04	20107	5305	946	47	6192	136.09
Heine 07	263	169	17	5	179	44.68
Hoffmann 01	2974	1176	95	22	1247	59.38
Hoffmann 02	1076	534	29	11	549	54.90
Hoffmann 03	8163	2511	290	34	2759	83.61
Immermann 01	28943	6397	918	63	7234	116.68
Kafka 01	10256	2321	448	41	2717	67.93
Kafka 02	3181	1210	159	23	1343	62.47
Kafka 03	1072	513	34	12	532	46.98
Kafka 04	625	321	23	10	332	39.11
Kafka 05	247	166	14	5	173	43.17
Kafka 06	178	137	6	4	138	46.02
Kafka 07	132	89	9	4	93	35.03
Kafka 08	139	102	9	4	106	42.31
Kafka 09	596	343	25	9	358	44.73
Kafka 10	86	62	4	4	62	20.75
Kafka 11	151	104	9	5	107	30.61
Kafka 12	160	101	9	5	104	25.93
Kafka 13	232	150	9	6	153	30.54
Kafka 14	142	104	11	3	111	55.45
Kafka 15	189	136	7	5	138	39.40
Kafka 16	255	177	10	6	181	36.11
Kafka 17	111	80	11	3	86	43.08
Kafka 18	61	48	3	3	48	31.89
Kafka 19	41	33	3	2	33	32.83
Kafka 20	1402	539	74	15	596	43.37
Kafka 21	610	364	18	10	371	43.69

Kafka 22	2129	887	89	18	956	55.19
Kafka 23	255	153	13	6	159	31.74
Kafka 24	584	276	25	9	290	38.66
Kafka 25	3414	1214	104	23	1290	58.64
Kafka 26	134	98	7	4	100	40.16
Kafka 27	428	240	14	8	246	35.10
Kafka 28	470	272	13	8	277	39.56
Keller 01	25625	5516	1399	59	6840	117.93
Keller 02	301	196	20	5	209	52.13
Keller 03	13149	3512	724	43	4181	99.55
Keller 04	1896	897	103	15	980	70.01
Lessing 01	114	78	7	4	80	26.63
Lessing 02	208	141	13	4	148	49.50
Lessing 03	61	48	4	3	48	32.16
Lessing 04	47	41	2	2	40	40.41
Lessing 05	182	120	7	5	121	34.71
Lessing 06	362	227	13	7	232	38.63
Lessing 07	231	161	9	4	165	54.88
Lessing 08	74	64	4	2	65	64.65
Lessing 09	327	193	24	6	210	41.95
Lessing 10	254	154	12	6	159	31.78
Löns 01	1672	706	95	15	782	55.86
Löns 02	2988	928	141	23	1042	47.36
Löns 03	4063	1162	172	26	1303	52.12
Löns 04	3713	1081	167	24	1218	52.96
Löns 05	4676	1235	254	28	1457	53.96
Löns 06	4833	1364	244	29	1573	56.18
Löns 07	7743	1862	414	36	2232	63.77
Löns 08	6093	1724	328	31	2015	67.17
Löns 09	9252	2126	453	39	2531	66.61
Löns 10	6546	1736	274	35	1968	57.88
Löns 11	4102	1294	217	27	1481	56.96
Löns 12	4432	1318	221	26	1507	60.28
Löns 13	1361	556	60	14	600	46.15
Meyer 01	1523	801	56	14	840	64.62
Meyer 02	573	331	26	8	347	49.57
Meyer 03	1052	551	46	11	583	58.30
Meyer 04	2550	1142	79	18	1197	70.41

Meyer 05	1249	658	47	12	690	62.73
Meyer 06	833	471	34	10	492	54.67
Meyer 07	1229	652	47	13	683	56.92
Meyer 08	1028	556	43	11	585	58.50
Meyer 09	776	441	40	9	471	58.88
Meyer 10	940	493	41	11	520	52.00
Meyer 11	2398	1079	88	17	1146	71.63
Novalis 01	2894	1129	139	21	1243	62.15
Novalis 02	3719	1487	208	22	1669	79.48
Novalis 03	5321	1819	233	25	2018	84.08
Novalis 04	2777	1282	130	18	1389	81.71
Novalis 05	8866	2769	473	35	3198	94.06
Novalis 06	4030	1467	178	23	1617	73.50
Novalis 07	1744	792	77	16	851	56.73
Novalis 08	2111	816	75	17	869	54.31
Novalis 09	8945	2681	442	32	3082	99.42
Novalis 10	5367	1939	238	26	2144	85.76
Novalis 11	1358	646	83	12	714	66.96
Novalis 12	4430	1697	195	24	1861	80.91
Novalis 13	1080	514	58	12	557	49.14
Paul 01	854	487	37	10	512	56.89
Paul 02	383	255	14	6	260	52.00
Paul 03	520	311	26	8	326	46.57
Paul 04	580	354	21	8	365	52.14
Paul 05	1331	677	44	12	705	64.09
Paul 06	526	305	16	8	313	44.71
Paul 07	508	316	15	7	323	53.83
Paul 08	402	248	22	6	262	52.40
Paul 09	1068	547	37	10	570	63.33
Paul 10	1558	778	53	13	814	67.83
Paul 11	2232	1027	84	15	1092	78.00
Paul 12	620	365	25	8	380	54.29
Paul 13	1392	652	40	13	676	56.33
Paul 14	1400	714	49	14	746	57.38
Paul 15	1648	793	65	15	840	60.00
Paul 16	320	223	12	5	227	56.75
Paul 17	1844	897	73	15	952	68.00
Paul 18	870	489	42	11	520	52.00

Paul 19	1236	676	38	13	699	58.25
Paul 20	2059	1011	78	16	1068	71.20
Paul 21	3955	1513	172	24	1659	72.13
Paul 22	478	302	15	7	309	51.50
Paul 23	656	386	26	9	401	50.13
Paul 24	1465	730	80	13	795	66.25
Paul 25	588	361	18	8	370	52.86
Paul 26	1896	887	61	15	930	66.43
Paul 27	749	410	26	9	426	53.25
Paul 28	241	172	8	5	174	43.50
Paul 29	1825	872	68	14	921	70.85
Paul 30	388	238	17	6	248	49.60
Paul 31	1630	753	72	14	810	62.31
Paul 32	163	119	6	4	120	40.00
Paul 33	596	355	23	8	369	52.71
Paul 35	1947	897	82	17	960	60.00
Paul 36	425	253	15	7	259	43.17
Paul 37	368	239	12	6	243	48.60
Paul 38	1218	636	40	12	660	60.00
Paul 39	388	248	13	7	253	42.17
Paul 40	1370	655	53	14	694	53.38
Paul 41	1032	546	43	11	575	57.50
Paul 42	1546	731	50	13	764	63.67
Paul 43	4148	1591	152	26	1714	68.56
Paul 44	1881	896	66	15	943	67.36
Paul 45	2723	1102	155	18	1236	72.71
Paul 46	3095	1276	99	21	1351	67.55
Paul 47	516	319	19	8	330	47.14
Paul 48	1200	604	50	13	638	53.17
Paul 49	562	336	19	8	346	49.43
Paul 50	430	255	23	7	269	44.83
Paul 51	3222	1323	116	20	1413	74.37
Paul 52	1731	815	71	15	870	62.14
Paul 53	1839	864	75	14	922	70.92
Paul 54	6644	2417	245	30	2625	90.52
Paul 55	7854	2680	321	33	2961	92.53
Paul 56	963	482	47	10	516	57.33
Pseudonym 01	728	363	30	10	381	42.33

Pseudonym 02	612	326	23	9	339	42.38
Raabe 01	13045	3003	691	45	3638	82.68
Raabe 02	3173	962	134	23	1070	48.64
Raabe 03	2690	950	135	21	1060	53.00
Raabe 04	6253	2110	282	30	2355	81.21
Raabe 05	5087	1801	196	26	1964	78.56
Rieder 01	1161	510	36	12	532	48.36
Rieder 02	1231	472	55	13	511	42.58
Rückert 01	141	97	10	4	102	33.87
Rückert 02	327	202	9	7	205	34.12
Rückert 03	152	107	8	4	110	36.57
Rückert 04	721	412	22	9	423	52.86
Rückert 05	212	145	10	5	149	37.30
Schnitzler 01	2793	961	109	20	1044	56.43
Schnitzler 02	1936	825	59	17	864	54.02
Schnitzler 03	801	410	28	11	425	42.54
Schnitzler 04	2489	870	135	21	982	49.93
Schnitzler 05	2123	822	110	18	910	54.57
Schnitzler 06	1539	668	50	15	701	51.89
Schnitzler 07	5652	1451	259	31	1673	55.31
Schnitzler 08	1711	666	63	15	711	52.17
Schnitzler 09	6552	1993	207	32	2161	70.32
Schnitzler 10	1349	629	49	15	661	48.95
Schnitzler 11	1595	723	97	15	803	57.34
Schnitzler 12	6173	1476	400	31	1835	61.17
Schnitzler 13	1184	544	44	13	573	47.72
Schnitzler 14	3900	1309	139	26	1415	57.76
Sealsfield 01	1352	600	45	13	629	52.42
Sealsfield 02	4663	1825	142	27	1936	74.46
Sealsfield 03	3238	1197	114	21	1284	64.20
Sealsfield 04	3954	1399	161	24	1530	66.52
Sealsfield 05	3187	1079	96	22	1149	54.71
Sealsfield 06	2586	1010	67	20	1053	55.42
Sealsfield 07	2939	1035	75	20	1086	57.16
Sealsfield 08	4865	1333	138	27	1435	55.19
Sealsfield 09	7259	2295	263	31	2519	83.97
Sealsfield 10	4838	1620	138	26	1726	69.04
Sealsfield 11	3785	1265	98	26	1333	53.32

Sealsfield 12	3019	1191	95	20	1262	66.42
Sealsfield 13	2370	1071	89	17	1139	71.19
Sealsfield 14	2744	1198	82	19	1257	69.83
Sealsfield 15	4786	1545	164	27	1676	64.46
Sealsfield 16	4497	1602	137	26	1707	68.28
Sealsfield 17	6705	2273	192	30	2429	83.76
Sealsfield 18	4162	1252	285	24	1508	65.57
Sealsfield 19	5626	1653	171	29	1789	63.89
Sealsfield 20	8423	2735	273	35	2966	87.24
Sealsfield 21	6041	2040	220	29	2224	79.43
Sealsfield 22	5748	1655	157	29	1776	63.43
Sealsfield 23	1752	799	80	14	861	66.23
Sealsfield 24	1696	753	68	14	803	61.77
Sealsfield 25	1368	704	40	12	730	66.36
Sealsfield 26	1517	679	44	15	706	50.43
Sealsfield 27	4195	1516	179	24	1665	72.39
Sealsfield 28	1515	586	70	15	636	45.43
Storm 01	38306	6233	1292	76	7427	99.03
Sudermann 01	11437	2427	507	43	2879	68.55
Tucholsky 01	8544	2449	351	35	2757	81.09
Tucholsky 02	7106	1935	207	35	2100	61.76
Tucholsky 03	9699	2502	336	38	2790	75.41
Tucholsky 04	7415	1968	214	35	2139	62.91
Tucholsky 05	4823	1399	174	28	1537	56.93
Wedekind 01	4035	1336	122	26	1428	57.12
Wedekind 02	6040	1731	179	31	1872	62.40
Wedekind 03	7402	1934	276	34	2168	65.70
Wedekind 04	1297	646	44	13	676	56.33
Wedekind 05	1935	580	89	19	645	35.85
Wedekind 06	5955	1689	249	34	1901	57.62
Wedekind 07	605	341	22	9	352	44.02
Wedekind 08	2033	855	87	17	921	57.55

As can be seen, even the variation within the works of one author may be considerable but the sample sizes are so large that the confidence intervals are relatively small. The strong dependence of the shape ratio on text size is shown in Figure 7.10.

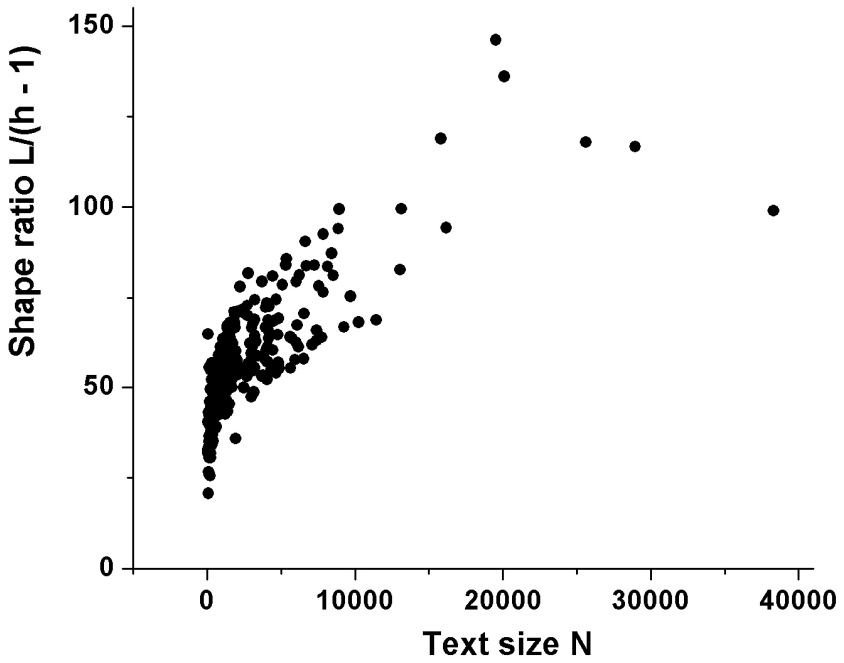


Figure 7.10. The course of the shape ratio of 253 German texts in terms of text size

Finally, Table 7.9 presents the investigated 26 German writers ranked in increasing order of the mean shape ratio.

Table 7.9
The mean shape ratio of 26 German writers
(increasing order)

mid life year	author	mean *L*/(*h* - 1)	stdev L/(h - 1)	number of texts
1827	Rückert	38.94	7.92	5
1904	Kafka	41.44	10.88	28
1755	Lessing	41.53	11.73	10
2001	pseudonym	42.35	0.03	2
2001	Rieder	45.47	4.09	2
1791	Goethe	46.85	16.12	8
1897	Schnitzler	54.29	6.62	14
1891	Wedekind	54.57	9.82	8
1810	Chamisso	56.37	4.66	11
1890	Löns	56.71	6.54	13
1823	Eichendorff	56.93	4.17	10

1794	Paul	58.99	11.22	56
1862	Meyer	59.84	6.99	11
1823	Droste	61.64	17.00	6
1806	Arnim	63.19	13.48	3
1829	Sealsfield	65.45	10.38	28
1799	Hoffmann	65.96	15.44	3
1913	Tucholsky	67.62	10.16	5
1893	Sudermann	68.55		1
1871	Raabe	68.82	16.56	5
1787	Novalis	74.48	15.59	13
1827	Heine	83.45	53.06	5
1855	Keller	84.91	29.45	4
1853	Storm	99.03		1
1818	Immermann	116.68		1
1870	Busch	118.88		1

However, there is no time-trend in the shape ratio, as can be seen in Figure 7.11. It may be expected that the analysis of further German texts would yield rather a cloud of points than a clearer trend. This is, however, a good argument agaist the assumption that German develops towards analyticity as did English. It must be borne in mind that examples of recent changes to analytism taken in isolation do not furnish any evidence if the language is not analyzed as a whole. However, a simple indicator taking complete texts may be more illuminating.

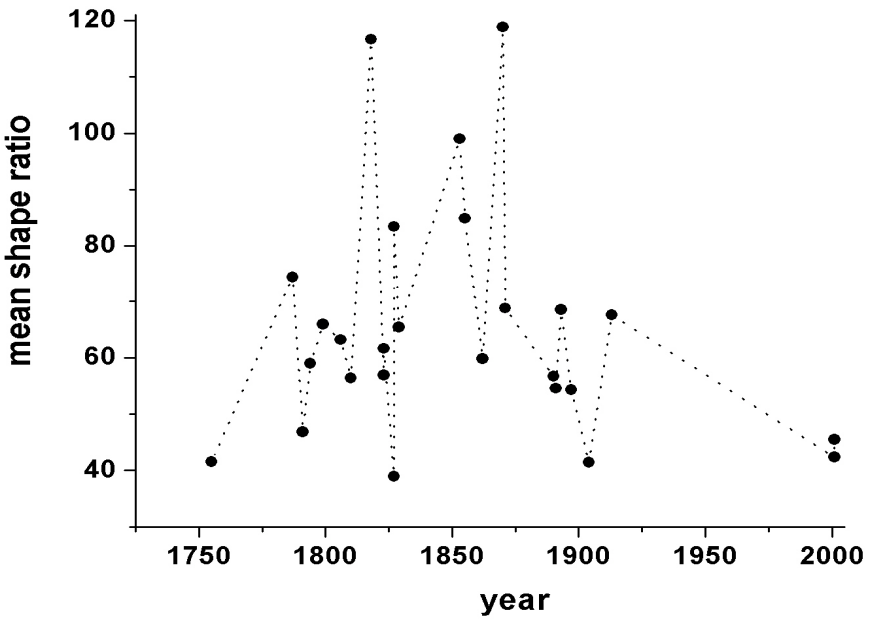


Figure 7.11. The mean shape ratio for German texts according to years

Last but not least, we introduce a slightly modified Hirsch coefficient, *a*. It should be noted that this coefficient has been defined (Hirsch 2005) as the proportionality constant between the sample size *N* and the square of the *h*-point coordinate by the relationship $N = ah^2$. However, since *N* is the area under the rank-frequency sequence defined by the points (1, *f*(1)), (1, 1), and (*R*, 1) whose zero-point is (1,1) and not (0,0), and the natural *h*-square unit is defined by the points (1, 1), (1, *h*), (*h*, *h*), and (*h*, 1), and has the area $(h - 1)^2$, it is more appropriate to rewrite the relationship $N = ah^2$ as

(7.12) $N = b(h - 1)^2$

where the old indicator *a* and the new one *b* are related by

(7.13) $a/b = (1 - 1/h)^2$

Considering further expression (7.12) and the indicator *p* (5.26) from Chapter 5.3 as given by

(7.14) $p = \dfrac{L_{\max} - L}{h - 1}$

another indicator can be established, namely

(7.15) $q = \dfrac{L_{max} - L}{N^{1/2}}$,

however $N^{1/2}$ being taken into account instead of $(h - 1)$. Since both L_{max} and *L* increase with increasing *N*, the increase is made in this way relative. Finally, the three indicators *b*, *p*, *q* are connected by the simple relation

(7.16) $b = \dfrac{N}{(h-1)^2} = \left(\dfrac{p}{q}\right)^2$.

Unlike *p*, the new *q*-indicator is able to express both differences between individual texts and differences between languages if the means are taken. For this purpose, let us consider first the individual values in 100 texts from 20 languages (cf. Table 7.10). The values of *p*, *q*, *b* have been computed according to (7.14), (7.15), (7.16) respectively.

Table 7.10
Indicators (7.14), (7.15) and (7.16) in 100 texts from 20 languages
(The texts were taken from Popescu et al. 2009)

ID	N	V	$f(1)$	h	L	L_{max}	p	q	b
B 01	761	400	40	10	428	438	1.111	0.362	9.395
B 02	352	201	13	8	205	212	1.000	0.373	7.184
B 03	515	285	15	9	290	298	1.000	0.353	8.047
B 04	483	286	21	8	297	305	1.143	0.364	9.857
B 05	406	238	19	7	247	255	1.333	0.397	11.278
Cz 01	1044	638	58	9	684	694	1.250	0.309	16.313
Cz 02	984	543	56	11	586	597	1.100	0.351	9.840
Cz 03	2858	1274	182	19	1432	1454	1.222	0.412	8.821
Cz 04	522	323	27	7	342	348	1.000	0.263	14.500
Cz 05	999	556	84	9	627	638	1.375	0.348	15.609
E 01	2330	939	126	16	1043	1063	1.333	0.414	10.356
E 02	2971	1017	168	22	1157	1183	1.238	0.477	6.737
E 03	3247	1001	229	19	1205	1228	1.278	0.404	10.022
E 04	4622	1232	366	23	1567	1596	1.318	0.427	9.550
E 05	4760	1495	297	26	1761	1790	1.160	0.420	7.616
E 07	5004	1597	237	25	1801	1832	1.292	0.438	8.688
E 13	11265	1659	780	41	2388	2437	1.225	0.462	7.041
G 05	559	332	30	8	351	360	1.286	0.381	11.408
G 09	653	379	30	9	398	407	1.125	0.352	10.203
G 10	480	301	18	7	310	317	1.167	0.320	13.333
G 11	468	297	18	7	307	313	1.000	0.277	13.000
G 12	251	169	14	6	175	181	1.200	0.379	10.040
G 14	184	129	10	5	133	137	1.000	0.295	11.500
G 17	225	124	11	6	128	133	1.000	0.333	9.000
H 01	2044	1079	225	12	1289	1302	1.182	0.288	16.893
H 02	1288	789	130	8	907	917	1.429	0.279	26.286
H 03	403	291	48	4	332	337	1.667	0.249	44.778
H 04	936	609	76	7	674	683	1.500	0.294	26.000
H 05	413	290	32	6	314	320	1.200	0.295	16.520
Hw 03	3507	521	277	26	764	796	1.280	0.540	5.611
Hw 04	7892	744	535	38	1229	1277	1.297	0.540	5.765
Hw 05	7620	680	416	38	1047	1094	1.270	0.538	5.566
Hw 06	12356	1039	901	44	1877	1938	1.419	0.549	6.683

I 01	11760	3667	388	37	4007	4053	1.278	0.424	9.074
I 02	6064	2203	257	25	2426	2458	1.333	0.411	10.528
I 03	854	483	64	10	534	545	1.222	0.376	10.543
I 04	3258	1237	118	21	1330	1353	1.150	0.403	8.145
I 05	1129	512	42	12	537	552	1.364	0.446	9.331
In 01	376	221	16	6	228	235	1.400	0.361	15.040
In 02	373	209	18	7	219	225	1.000	0.311	10.361
In 03	347	194	14	6	200	206	1.200	0.322	13.880
In 04	343	213	11	5	217	222	1.250	0.270	21.438
In 05	414	188	16	8	196	202	0.857	0.295	8.449
Kn 003	3188	1833	74	13	1891	1905	1.167	0.248	22.139
Kn 004	1050	720	23	7	733	741	1.333	0.247	29.167
Kn 005	4869	2477	101	16	2558	2576	1.200	0.258	21.640
Kn 006	5231	2433	74	20	2481	2505	1.263	0.332	14.490
Kn 011	4541	2516	63	17	2558	2577	1.188	0.282	17.738
Lk 01	345	174	20	8	185	192	1.000	0.377	7.041
Lk 02	1633	479	124	17	580	601	1.313	0.520	6.379
Lk 03	809	272	62	12	318	332	1.273	0.492	6.686
Lk 04	219	116	18	6	126	132	1.200	0.405	8.760
Lt 01	3311	2211	133	12	2328	2342	1.273	0.243	27.364
Lt 02	4010	2334	190	18	2502	2522	1.176	0.316	13.875
Lt 03	4931	2703	103	19	2783	2804	1.167	0.299	15.219
Lt 04	4285	1910	99	20	1983	2007	1.263	0.367	11.870
Lt 05	1354	909	33	8	930	940	1.429	0.272	27.633
Lt 06	829	609	19	7	621	626	0.833	0.174	23.028
M 01	2062	398	152	18	527	548	1.235	0.462	7.135
M 02	1175	277	127	15	386	402	1.143	0.467	5.995
M 03	1434	277	128	17	385	403	1.125	0.475	5.602
M 04	1289	326	137	15	444	461	1.214	0.474	6.577
M 05	3620	514	234	26	715	746	1.240	0.515	5.792
Mq 01	2330	289	247	22	507	534	1.286	0.559	5.283
Mq 02	457	150	42	10	179	190	1.222	0.515	5.642
Mq 03	1509	301	218	14	500	517	1.308	0.438	8.929
Mr 001	2998	1555	75	14	1612	1628	1.231	0.292	17.740
Mr 018	4062	1788	126	20	1890	1912	1.158	0.345	11.252
Mr 026	4146	2038	84	19	2099	2120	1.167	0.326	12.796
Mr 027	4128	1400	92	21	1468	1490	1.100	0.342	10.320
Mr 288	4060	2079	84	17	2141	2161	1.250	0.314	15.859

R 01	1738	843	62	14	886	903	1.308	0.408	10.284
R 02	2279	1179	110	16	1269	1287	1.200	0.377	10.129
R 03	1264	719	65	12	770	782	1.091	0.338	10.446
R 04	1284	729	49	10	764	776	1.333	0.335	15.852
R 05	1032	567	46	11	599	611	1.200	0.374	10.320
R 06	695	432	30	10	452	460	0.889	0.303	8.580
Rt 01	968	223	111	14	316	332	1.231	0.514	5.728
Rt 02	845	214	69	13	265	281	1.333	0.550	5.868
Rt 03	892	207	66	13	256	271	1.250	0.502	6.194
Rt 04	625	181	49	11	216	228	1.200	0.480	6.250
Rt 05	1059	197	74	15	251	269	1.286	0.553	5.403
Ru 01	753	422	31	8	441	451	1.429	0.364	15.367
Ru 02	2595	1240	138	16	1357	1376	1.267	0.373	11.533
Ru 03	3853	1792	144	21	1909	1934	1.250	0.403	9.633
Ru 04	6025	2536	228	25	2732	2762	1.250	0.386	10.460
Ru 05	17205	6073	701	41	6722	6772	1.250	0.381	10.753
Sl 01	756	457	47	9	494	502	1.000	0.291	11.813
Sl 02	1371	603	66	13	651	667	1.333	0.432	9.521
Sl 03	1966	907	102	13	991	1007	1.333	0.361	13.653
Sl 04	3491	1102	328	21	1404	1428	1.200	0.406	8.728
Sl 05	5588	2223	193	25	2385	2414	1.208	0.388	9.701
Sm 01	1487	267	159	17	403	424	1.313	0.545	5.809
Sm 02	1171	222	103	15	304	323	1.357	0.555	5.974
Sm 03	617	140	45	13	168	183	1.250	0.604	4.285
Sm 04	736	153	78	12	214	229	1.364	0.553	6.083
Sm 05	447	124	39	11	149	161	1.200	0.568	4.470
T 01	1551	611	89	14	681	698	1.308	0.432	9.178
T 02	1827	720	107	15	807	825	1.286	0.421	9.321
T 03	2054	645	128	19	749	771	1.222	0.485	6.340

The variability and independence of p and q in terms of the text size N is illustrated in Figure 7.12.

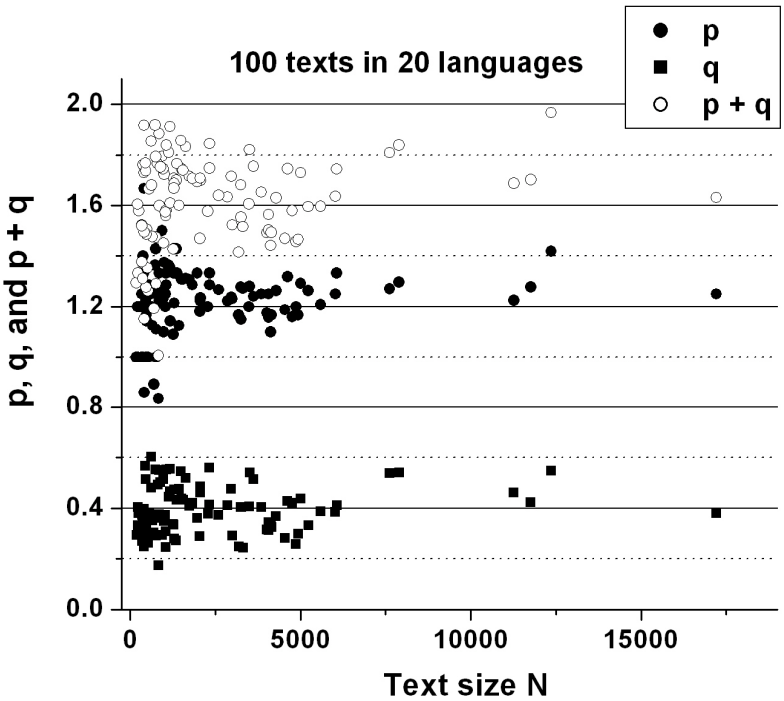


Figure 7.12. *p*, *q*, and their sum in terms of the text size *N* for the data of Table 7.10

Another single-language example is presented in Table 7.11 showing the individual *p*, *q*, *b* values of 253 texts of 26 German writers.

Table 7.11
Indicators *p*, *q*, *b* in 253 texts of 26 German writers
(see authors and text titles in Appendix)

ID	*N*	*V*	*f*(1)	*h*	*L*	L_{max}	*p*	*q*	*b*
Arnim 01	7846	2221	271	33	2448	2490	1.313	0.474	7.662
Arnim 02	1201	564	46	13	595	608	1.123	0.389	8.340
Arnim 03	4167	1429	189	26	1588	1616	1.120	0.434	6.667
Busch 01	15820	4642	527	44	5112	5167	1.279	0.437	8.556
Chamisso 01	2210	884	82	18	944	964	1.176	0.425	7.647
Chamisso 02	1847	808	84	16	872	890	1.200	0.419	8.209
Chamisso 03	1428	630	70	14	684	698	1.077	0.370	8.450
Chamisso 04	3205	1209	123	20	1305	1330	1.316	0.442	8.878
Chamisso 05	2108	853	79	18	911	930	1.118	0.414	7.294
Chamisso 06	1948	801	75	17	853	874	1.313	0.476	7.609
Chamisso 07	1362	670	44	13	698	712	1.167	0.379	9.458

Chamisso 08	1870	788	80	16	848	866	1.200	0.416	8.311
Chamisso 09	1320	593	96	14	673	687	1.077	0.385	7.811
Chamisso 10	1012	536	52	11	575	586	1.100	0.346	10.120
Chamisso 11	1386	656	66	14	705	720	1.154	0.403	8.201
Droste 01	16172	4064	525	49	4528	4587	1.229	0.464	7.019
Droste 02	884	492	48	10	527	538	1.275	0.370	11.897
Droste 03	700	425	31	9	444	454	1.240	0.375	10.938
Droste 04	786	408	34	11	430	440	1.084	0.367	8.709
Droste 05	1274	657	51	13	692	706	1.216	0.392	9.633
Droste 08	965	509	39	11	535	546	1.145	0.369	9.650
Eichendorff 01	3080	1079	177	21	1228	1254	1.300	0.468	7.700
Eichendorff 02	4100	1287	210	25	1466	1495	1.208	0.453	7.118
Eichendorff 03	4342	1334	182	28	1482	1514	1.185	0.486	5.956
Eichendorff 04	1781	739	79	16	799	816	1.133	0.403	7.916
Eichendorff 05	1680	699	70	16	750	767	1.133	0.415	7.467
Eichendorff 06	3223	1059	130	22	1163	1187	1.143	0.423	7.308
Eichendorff 07	2594	932	121	20	1031	1051	1.053	0.393	7.186
Eichendorff 08	3987	1320	159	25	1447	1477	1.250	0.475	6.922
Eichendorff 09	3285	1185	155	22	1315	1338	1.095	0.401	7.449
Eichendorff 10	3052	1073	131	22	1178	1202	1.143	0.434	6.921
Goethe 01	7554	2222	318	33	2502	2538	1.125	0.414	7.377
Goethe 05	559	332	30	8	351	360	1.286	0.381	11.408
Goethe 09	653	379	30	9	398	407	1.125	0.352	10.203
Goethe 10	480	301	18	7	310	317	1.167	0.320	13.333
Goethe 11	468	297	18	7	307	313	1.000	0.277	13.000
Goethe 12	251	169	14	6	175	181	1.200	0.379	10.040
Goethe 14	184	129	10	5	133	137	1.000	0.295	11.500
Goethe 17	225	124	11	6	128	133	1.000	0.333	9.000
Heine 01	19522	5769	939	47	6648	6706	1.275	0.415	9.430
Heine 02	603	361	50	9	400	409	1.171	0.358	10.720
Heine 03	394	211	21	7	222	230	1.302	0.393	10.944
Heine 04	20107	5305	946	47	6192	6249	1.253	0.402	9.712
Heine 07	263	169	17	5	179	184	1.320	0.326	16.438
Hoffmann 01	2974	1176	95	22	1247	1269	1.048	0.403	6.744
Hoffmann 02	1076	534	29	11	549	561	1.200	0.366	10.760
Hoffmann 03	8163	2511	290	34	2759	2799	1.212	0.443	7.496
Immermann 01	28943	6397	918	63	7234	7313	1.274	0.464	7.529
Kafka 01	10256	2321	448	41	2717	2767	1.250	0.494	6.410

Kafka 02	3181	1210	159	23	1343	1367	1.116	0.426	6.882
Kafka 03	1072	513	34	12	532	545	1.123	0.388	8.351
Kafka 04	625	321	23	10	332	342	1.121	0.381	8.651
Kafka 05	247	166	14	5	173	178	1.333	0.339	15.438
Kafka 06	178	137	6	4	138	141	0.977	0.220	19.778
Kafka 07	132	89	9	4	93	96	1.056	0.245	18.656
Kafka 08	139	102	9	4	106	109	1.288	0.273	22.240
Kafka 09	596	343	25	9	358	366	1.025	0.336	9.313
Kafka 10	86	62	4	4	62	64	0.587	0.190	9.556
Kafka 11	151	104	9	5	107	111	1.106	0.315	12.327
Kafka 12	160	101	9	5	104	108	1.070	0.338	10.000
Kafka 13	232	150	9	6	153	157	0.856	0.281	9.280
Kafka 14	142	104	11	3	111	113	1.055	0.177	35.500
Kafka 15	189	136	7	5	138	141	0.889	0.226	15.429
Kafka 16	255	177	10	6	181	185	0.892	0.279	10.200
Kafka 17	111	80	11	3	86	89	1.425	0.271	27.750
Kafka 18	61	48	3	3	48	49	0.780	0.150	27.111
Kafka 19	41	33	3	2	33	34	1.170	0.183	41.000
Kafka 20	1402	539	74	15	596	611	1.065	0.391	7.416
Kafka 21	610	364	18	10	371	380	1.015	0.349	8.443
Kafka 22	2129	887	89	18	956	974	1.012	0.380	7.089
Kafka 23	255	153	13	6	159	164	1.058	0.331	10.200
Kafka 24	584	276	25	9	290	299	1.204	0.374	10.382
Kafka 25	3414	1214	104	23	1290	1316	1.182	0.445	7.054
Kafka 26	134	98	7	4	100	103	1.040	0.225	21.440
Kafka 27	428	240	14	8	246	252	0.899	0.304	8.735
Kafka 28	470	272	13	8	277	283	0.873	0.282	9.592
Keller 01	25625	5516	1399	59	6840	6913	1.259	0.456	7.617
Keller 02	301	196	20	5	209	214	1.370	0.316	18.813
Keller 03	13149	3512	724	43	4181	4234	1.262	0.462	7.454
Keller 04	1896	897	103	15	980	998	1.273	0.409	9.673
Lessing 01	114	78	7	4	80	83	1.037	0.291	12.667
Lessing 02	208	141	13	4	148	152	1.170	0.243	23.111
Lessing 03	61	48	4	3	48	50	1.173	0.225	27.111
Lessing 04	47	41	2	2	40	41	0.590	0.086	47.000
Lessing 05	182	120	7	5	121	125	1.003	0.260	14.857
Lessing 06	362	227	13	7	232	238	1.035	0.326	10.056
Lessing 07	231	161	9	4	165	168	1.120	0.221	25.667

Lessing 08	74	64	4	2	65	66	1.350	0.157	74.000
Lessing 09	327	193	24	6	210	215	1.050	0.290	13.080
Lessing 10	254	154	12	6	159	164	1.024	0.321	10.160
Löns 01	1672	706	95	15	782	799	1.214	0.416	8.531
Löns 02	2988	928	141	23	1042	1067	1.136	0.457	6.174
Löns 03	4063	1162	172	26	1303	1332	1.160	0.455	6.501
Löns 04	3713	1081	167	24	1218	1246	1.217	0.460	7.019
Löns 05	4676	1235	254	28	1457	1487	1.111	0.439	6.414
Löns 06	4833	1364	244	29	1573	1606	1.179	0.475	6.165
Löns 07	7743	1862	414	36	2232	2274	1.200	0.477	6.321
Löns 08	6093	1724	328	31	2015	2050	1.167	0.448	6.770
Löns 09	9252	2126	453	39	2531	2577	1.211	0.478	6.407
Löns 10	6546	1736	274	35	1968	2008	1.176	0.494	5.663
Löns 11	4102	1294	217	27	1481	1509	1.077	0.437	6.068
Löns 12	4432	1318	221	26	1507	1537	1.200	0.451	7.091
Löns 13	1361	556	60	14	600	614	1.077	0.379	8.053
Meyer 01	1523	801	56	14	840	855	1.154	0.384	9.012
Meyer 02	573	331	26	8	347	355	1.143	0.334	11.694
Meyer 03	1052	551	46	11	583	595	1.200	0.370	10.520
Meyer 04	2550	1142	79	18	1197	1219	1.294	0.436	8.824
Meyer 05	1249	658	47	12	690	703	1.182	0.368	10.322
Meyer 06	833	471	34	10	492	503	1.222	0.381	10.284
Meyer 07	1229	652	47	13	683	697	1.167	0.399	8.535
Meyer 08	1028	556	43	11	585	597	1.200	0.374	10.280
Meyer 09	776	441	40	9	471	479	1.000	0.287	12.125
Meyer 10	940	493	41	11	520	532	1.200	0.391	9.400
Meyer 11	2398	1079	88	17	1146	1165	1.188	0.388	9.367
Novalis 01	2894	1129	139	21	1243	1266	1.150	0.428	7.235
Novalis 02	3719	1487	208	22	1669	1693	1.143	0.394	8.433
Novalis 03	5321	1819	233	25	2018	2050	1.333	0.439	9.238
Novalis 04	2777	1282	130	18	1389	1410	1.235	0.399	9.609
Novalis 05	8866	2769	473	35	3198	3240	1.235	0.446	7.670
Novalis 06	4030	1467	178	23	1617	1643	1.182	0.410	8.326
Novalis 07	1744	792	77	16	851	867	1.067	0.383	7.751
Novalis 08	2111	816	75	17	869	889	1.250	0.435	8.246
Novalis 09	8945	2681	442	32	3082	3121	1.258	0.412	9.308
Novalis 10	5367	1939	238	26	2144	2175	1.240	0.423	8.587
Novalis 11	1358	646	83	12	714	727	1.235	0.357	11.950

Novalis 12	4430	1697	195	24	1861	1890	1.264	0.437	8.374
Novalis 13	1080	514	58	12	557	570	1.171	0.404	8.413
Paul 01	854	487	37	10	512	522	1.111	0.342	10.543
Paul 02	383	255	14	6	260	267	1.400	0.358	15.320
Paul 03	520	311	26	8	326	335	1.286	0.395	10.612
Paul 04	580	354	21	8	365	373	1.143	0.332	11.837
Paul 05	1331	677	44	12	705	719	1.273	0.384	11.000
Paul 06	526	305	16	8	313	319	0.857	0.262	10.735
Paul 07	508	316	15	7	323	329	1.000	0.266	14.111
Paul 08	402	248	22	6	262	268	1.200	0.299	16.080
Paul 09	1068	547	37	10	570	582	1.333	0.367	13.185
Paul 10	1558	778	53	13	814	829	1.250	0.380	10.819
Paul 11	2232	1027	84	15	1092	1109	1.214	0.360	11.388
Paul 12	620	365	25	8	380	388	1.143	0.321	12.653
Paul 13	1392	652	40	13	676	690	1.167	0.375	9.667
Paul 14	1400	714	49	14	746	761	1.154	0.401	8.284
Paul 15	1648	793	65	15	840	856	1.143	0.394	8.408
Paul 16	320	223	12	5	227	233	1.500	0.335	20.000
Paul 17	1844	897	73	15	952	968	1.143	0.373	9.408
Paul 18	870	489	42	11	520	529	0.900	0.305	8.700
Paul 19	1236	676	38	13	699	712	1.083	0.370	8.583
Paul 20	2059	1011	78	16	1068	1087	1.267	0.419	9.151
Paul 21	3955	1513	172	24	1659	1683	1.043	0.382	7.476
Paul 22	478	302	15	7	309	315	1.000	0.274	13.278
Paul 23	656	386	26	9	401	410	1.125	0.351	10.250
Paul 24	1465	730	80	13	795	808	1.083	0.340	10.174
Paul 25	588	361	18	8	370	377	1.000	0.289	12.000
Paul 26	1896	887	61	15	930	946	1.143	0.367	9.673
Paul 27	749	410	26	9	426	434	1.000	0.292	11.703
Paul 28	241	172	8	5	174	178	1.000	0.258	15.063
Paul 29	1825	872	68	14	921	938	1.308	0.398	10.799
Paul 30	388	238	17	6	248	253	1.000	0.254	15.520
Paul 31	1630	753	72	14	810	823	1.000	0.322	9.645
Paul 32	163	119	6	4	120	123	1.000	0.235	18.111
Paul 33	596	355	23	8	369	376	1.000	0.287	12.163
Paul 35	1947	897	82	17	960	977	1.063	0.385	7.605
Paul 36	425	253	15	7	259	266	1.167	0.340	11.806
Paul 37	368	239	12	6	243	249	1.200	0.313	14.720

Paul 38	1218	636	40	12	660	674	1.273	0.401	10.066
Paul 39	388	248	13	7	253	259	1.000	0.305	10.778
Paul 40	1370	655	53	14	694	706	0.923	0.324	8.107
Paul 41	1032	546	43	11	575	587	1.200	0.374	10.320
Paul 42	1546	731	50	13	764	779	1.250	0.381	10.736
Paul 43	4148	1591	152	26	1714	1741	1.080	0.419	6.637
Paul 44	1881	896	66	15	943	960	1.214	0.392	9.597
Paul 45	2723	1102	155	18	1236	1255	1.118	0.364	9.422
Paul 46	3095	1276	99	21	1351	1373	1.100	0.395	7.738
Paul 47	516	319	19	8	330	336	0.857	0.264	10.531
Paul 48	1200	604	50	13	638	652	1.167	0.404	8.333
Paul 49	562	336	19	8	346	353	1.000	0.295	11.469
Paul 50	430	255	23	7	269	276	1.167	0.338	11.944
Paul 51	3222	1323	116	20	1413	1437	1.263	0.423	8.925
Paul 52	1731	815	71	15	870	884	1.000	0.336	8.832
Paul 53	1839	864	75	14	922	937	1.154	0.350	10.882
Paul 54	6644	2417	245	30	2625	2660	1.207	0.429	7.900
Paul 55	7854	2680	321	33	2961	2999	1.188	0.429	7.670
Paul 56	963	482	47	10	516	527	1.222	0.354	11.889
Pseudonym 01	728	363	30	10	381	391	1.111	0.371	8.988
Pseudonym 02	612	326	23	9	339	347	1.000	0.323	9.563
Raabe 01	13045	3003	691	45	3638	3692	1.227	0.473	6.738
Raabe 02	3173	962	134	23	1070	1094	1.091	0.426	6.556
Raabe 03	2690	950	135	21	1060	1083	1.150	0.443	6.725
Raabe 04	6253	2110	282	30	2355	2390	1.207	0.443	7.435
Raabe 05	5087	1801	196	26	1964	1995	1.240	0.435	8.139
Rieder 01	1161	510	36	12	532	544	1.091	0.352	9.595
Rieder 02	1231	472	55	13	511	525	1.167	0.399	8.549
Rückert 01	141	97	10	4	102	105	1.133	0.286	15.667
Rückert 02	327	202	9	7	205	209	0.713	0.237	9.083
Rückert 03	152	107	8	4	110	113	1.097	0.267	16.889
Rückert 04	721	412	22	9	423	432	1.138	0.339	11.266
Rückert 05	212	145	10	5	149	153	0.953	0.262	13.250
Schnitzler 01	2793	961	109	20	1044	1068	1.297	0.454	8.161
Schnitzler 02	1936	825	59	17	864	882	1.105	0.402	7.563
Schnitzler 03	801	410	28	11	425	436	1.057	0.373	8.010
Schnitzler 04	2489	870	135	21	982	1003	1.066	0.420	6.433
Schnitzler 05	2123	822	110	18	910	930	1.215	0.439	7.640

Schnitzler 06	1539	668	50	15	701	716	1.143	0.393	8.444
Schnitzler 07	5652	1451	259	31	1673	1708	1.157	0.466	6.177
Schnitzler 08	1711	666	63	15	711	727	1.210	0.398	9.224
Schnitzler 09	6552	1993	207	32	2161	2198	1.204	0.457	6.938
Schnitzler 10	1349	629	49	15	661	676	1.122	0.412	7.402
Schnitzler 11	1595	723	97	15	803	818	1.086	0.381	8.138
Schnitzler 12	6173	1476	400	31	1835	1874	1.300	0.496	6.859
Schnitzler 13	1184	544	44	13	573	586	1.111	0.387	8.222
Schnitzler 14	3900	1309	139	26	1415	1446	1.265	0.496	6.497
Sealsfield 01	1352	600	45	13	629	643	1.167	0.381	9.389
Sealsfield 02	4663	1825	142	27	1936	1965	1.115	0.425	6.898
Sealsfield 03	3238	1197	114	21	1284	1309	1.250	0.439	8.095
Sealsfield 04	3954	1399	161	24	1530	1558	1.217	0.445	7.474
Sealsfield 05	3187	1079	96	22	1149	1173	1.143	0.425	7.227
Sealsfield 06	2586	1010	67	20	1053	1075	1.158	0.433	7.163
Sealsfield 07	2939	1035	75	20	1086	1108	1.158	0.406	8.141
Sealsfield 08	4865	1333	138	27	1435	1469	1.308	0.487	7.197
Sealsfield 09	7259	2295	263	31	2519	2556	1.233	0.434	8.066
Sealsfield 10	4838	1620	138	26	1726	1756	1.200	0.431	7.741
Sealsfield 11	3785	1265	98	26	1333	1361	1.120	0.455	6.056
Sealsfield 12	3019	1191	95	20	1262	1284	1.158	0.400	8.363
Sealsfield 13	2370	1071	89	17	1139	1158	1.188	0.390	9.258
Sealsfield 14	2744	1198	82	19	1257	1278	1.167	0.401	8.469
Sealsfield 15	4786	1545	164	27	1676	1707	1.192	0.448	7.080
Sealsfield 16	4497	1602	137	26	1707	1737	1.200	0.447	7.195
Sealsfield 17	6705	2273	192	30	2429	2463	1.172	0.415	7.973
Sealsfield 18	4162	1252	285	24	1508	1535	1.174	0.419	7.868
Sealsfield 19	5626	1653	171	29	1789	1822	1.179	0.440	7.176
Sealsfield 20	8423	2735	273	35	2966	3006	1.176	0.436	7.286
Sealsfield 21	6041	2040	220	29	2224	2258	1.214	0.437	7.705
Sealsfield 22	5748	1655	157	29	1776	1810	1.214	0.448	7.332
Sealsfield 23	1752	799	80	14	861	877	1.231	0.382	10.367
Sealsfield 24	1696	753	68	14	803	819	1.231	0.389	10.036
Sealsfield 25	1368	704	40	12	730	742	1.091	0.324	11.306
Sealsfield 26	1517	679	44	15	706	721	1.071	0.385	7.740
Sealsfield 27	4195	1516	179	24	1665	1693	1.217	0.432	7.930
Sealsfield 28	1515	586	70	15	636	654	1.286	0.462	7.730
Storm 01	38306	6233	1292	76	7427	7523	1.280	0.490	6.810

Sudermann 01	11437	2427	507	43	2879	2932	1.262	0.496	6.484
Tucholsky 01	8544	2449	351	35	2757	2798	1.206	0.444	7.391
Tucholsky 02	7106	1935	207	35	2100	2140	1.176	0.475	6.147
Tucholsky 03	9699	2502	336	38	2790	2836	1.243	0.467	7.085
Tucholsky 04	7415	1968	214	35	2139	2180	1.206	0.476	6.414
Tucholsky 05	4823	1399	174	28	1537	1571	1.259	0.490	6.616
Wedekind 01	4035	1336	122	26	1428	1456	1.120	0.441	6.456
Wedekind 02	6040	1731	179	31	1872	1908	1.200	0.463	6.711
Wedekind 03	7402	1934	276	34	2168	2208	1.212	0.465	6.797
Wedekind 04	1297	646	44	13	676	688	1.000	0.333	9.007
Wedekind 05	1935	580	89	19	645	667	1.208	0.494	5.972
Wedekind 06	5955	1689	249	34	1901	1936	1.052	0.450	5.468
Wedekind 07	605	341	22	9	352	361	1.101	0.358	9.453
Wedekind 08	2033	855	87	17	921	940	1.197	0.425	7.941

The variability and independence of *p* and *q* in terms of the text size *N* for the data of Table 7.11 is illustrated in Figure 7.13.

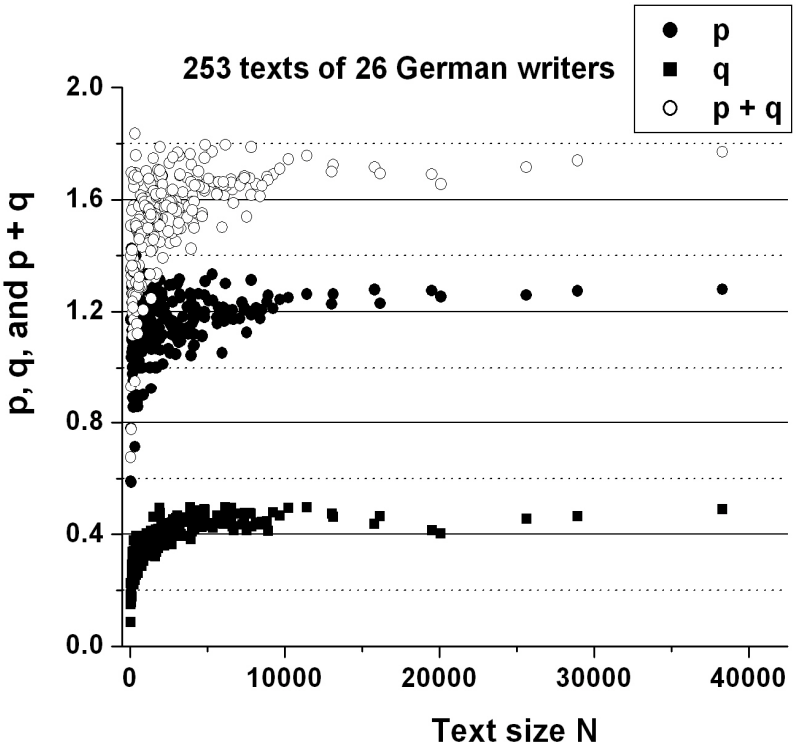


Figure 7.13. *p*, *q*, and their sum in terms of the text size *N* for the data of Table 7.11.

A general remark concerning the above data consists in the fact that, coincidentally or not, the sum of *p* and *q* appears to converge towards the golden number 1.618… with increasing N. More specifically, for the data of Table 7.10 we have the mean of (*p* + *q*) = 1.615 with the standard deviation of 0.183 whereas for the data of Table 7.11 we have the mean of (*p* + *q*) = 1.526 with the standard deviation of 0.176.

Since the variance of *L* is known (cf. Popescu, Mačutek, Altmann 2009), tests for differences of two *p* or *q* values are possible.

If we consider further the means of *p* we see that this indicator is not very adequate for typological purposes because differences between languages are not very conspicuous. However, if we consider *q* or *b*, we obtain a very clear picture. The smaller *q* or the greater *b*, the more synthetic is the language. Te results of averaging are presented in Table 7.12 where the languages are ordered according to $\bar{q}$.

Table 7.12
The mean indicators $\bar{p}$, $\bar{q}$, and $\bar{b}$ (ranked by $\bar{q}$)

Language	$\bar{p}$	$\bar{q}$	$\bar{b}$
Kannada	1.230	0.273	21.035
Latin	1.190	0.278	19.831
Hungarian	1.395	0.281	26.095
Indonesian	1.141	0.312	13.834
Marathi	1.181	0.324	13.593
German	1.111	0.334	11.212
Czech	1.189	0.336	13.017
Romanian	1.170	0.356	10.935
Bulgarian	1.117	0.370	9.152
Slovenian	1.215	0.376	10.683
Russian	1.289	0.382	11.549
Italian	1.269	0.412	9.524
English	1.263	0.435	8.573
Tagalog	1.272	0.446	8.279
Lakota	1.196	0.449	7.216
Maori	1.191	0.479	6.220
Marquesan	1.272	0.504	6.618
Rarotongan	1.260	0.520	5.889
Hawaiian	1.317	0.542	5.906
Samoan	1.297	0.565	5.324

8. Word frequency diversity and typology

In the previous chapters we saw that word form frequencies are characteristic of languages, at least in the sense that they signalize the position of language on the analytism/synthetism scale. Consequently, whatever operations we perform with word frequencies, they would reflect this property in some way.

In this chapter we shall use two well known indicators, namely the entropy (*H*) of word frequencies and the repeat rate (*RR*). We shall use the Shannon entropy but the study of other kinds of entropies whose number continuously increases (cf. Esteban, Morales 1995) would be interesting, too. Entropy is used in linguistics for different purposes; here it is used to characterize the dispersion (non uniformity) of frequencies. The Shannon entropy is defined as

$$(8.1) \quad H = -\sum_{r=1}^{V} p_r \log_2 p_r \ ,$$

where p_r is the relative frequency of word forms, i.e. $p_r = f_r/N$ for $r = 1,\ldots,V$. Replacing the relative frequencies by absolute ones we obtain

$$(8.2) \quad H = \log_2 N - \frac{1}{N}\sum_{r=1}^{V} f_r \log_2 f_r \ .$$

The variance of (8.1) is given as

$$(8.3) \quad Var(H) = \frac{1}{N}\left(\sum_{r=1}^{V} p_r \log_2^2 p_2 - H^2\right),$$

hence asymptotic tests are possible. Entropy lies in the interval <0, $\log_2 V$> and can be normalized to <0,1> using

$$(8.4) \quad H_{rel} = \frac{H}{log_2 V}.$$

The values of entropy and relative entropy in individual texts are presented in Table 8.1.

Table 8.1
Entropies of ranked word frequencies in 145 texts from 20 languages

Text	*N*	*V*	*H*	H_{rel}		Text	*N*	*V*	*H*	H_{rel}
B 01	761	400	7.8973	0.9136		Kn 18	4485	1782	9.7515	0.9030
B 02	352	201	7.0994	0.9279		Kn 19	1787	833	8.9712	0.9247
B 03	515	285	7.5827	0.9298		Kn 20	4556	1755	9.6909	0.8992
B 04	483	286	7.5980	0.9311		Kn 21	1455	790	8.9380	0.9286
B 05	406	238	7.3055	0.9254		Kn 22	4554	1794	9.6289	0.8908
B 06	687	388	7.8501	0.9128		Kn 23	4685	1738	9.6444	0.8961
B 07	557	324	7.7944	0.9346		Kn 30	4499	2005	10.0072	0.9123
B 08	268	179	7.1070	0.9496		Kn 31	4672	1920	9.8862	0.9064
B 09	550	313	7.6576	0.9237		Lk 01	345	174	6.7685	0.9094
B10	556	317	7.6055	0.9154		Lk 02	1633	479	7.3035	0.8203
Cz 01	1044	638	8.6163	0.9248		Lk 03	809	272	6.8508	0.8471
Cz 02	984	543	8.3282	0.9167		Lk 04	219	116	6.2882	0.9169
Cz 03	2858	1274	8.9529	0.8679		Lt 01	3311	2211	10.5032	0.9453
Cz 04	522	323	7.8770	0.9450		Lt 02	4010	2334	10.2814	0.9189
Cz 05	999	556	8.1959	0.8988		Lt 03	4931	2703	10.5934	0.9292
Cz 06	1612	840	8.6111	0.8864		Lt 04	4285	1910	9.8252	0.9014
Cz 07	2014	862	8.4876	0.8704		Lt 05	1354	909	9.3625	0.9526
Cz 08	677	389	7.9987	0.9297		Lt 06	829	609	8.4581	0.9144
Cz 09	460	259	7.4120	0.9246		M 01	2062	396	6.9856	0.8095
Cz 10	1156	638	8.4876	0.9109		M 02	1187	281	6.7198	0.8261
E 01	2330	939	8.5197	0.8628		M 03	1436	273	6.5851	0.8137
E 02	2971	1017	8.3972	0.8406		M 04	1409	302	6.6909	0.8122
E 03	3247	1001	8.2471	0.8274		M 05	3635	515	7.1346	0.7920
E 04	4622	1232	8.4634	0.8243		Mq 01	2330	289	6.6095	0.8085
E 05	4760	1495	8.7676	0.8314		Mq 02	451	143	6.1063	0.8529
E 06	4862	1176	8.2191	0.8058		Mq 03	1509	301	6.5012	0.7896
E 07	5004	1597	8.8057	0.8275		Mr 15	4693	1947	9.9538	0.9109
E 08	5083	985	7.9010	0.7946		Mr 16	3642	1831	9.8062	0.9048
E 09	5701	1574	8.6865	0.8179		Mr 17	4170	1853	10.0913	0.9296
E 10	6246	1333	8.3391	0.8033		Mr 18	4062	1788	10.6433	0.9851
E 11	8193	1669	8.5906	0.8025		Mr 20	3943	1725	10.4632	0.9731
E 12	9088	1825	8.5717	0.7912		Mr 21	3846	1793	10.1882	0.9426
E 13	11625	1659	8.4674	0.7916		Mr 22	4099	1703	10.3521	0.9644
G 01	1095	539	8.0326	0.8852		Mr 23	4142	1872	10.1542	0.9341
G 02	845	361	7.7006	0.9064		Mr 24	4255	1731	10.6589	0.9908
G 03	500	281	7.4369	0.9143		Mr 26	4146	2038	10.2964	0.9366

G 04	545	269	7.3530	0.9110		Mr 30	5054	2911	9.8764	0.8583
G 05	559	332	7.7183	0.9216		Mr 31	5105	2617	10.0120	0.8818
G 06	545	326	7.7918	0.9333		Mr 32	5195	2382	9.9799	0.8896
G 07	263	169	6.9781	0.9429		Mr 33	4339	2217	9.7898	0.8808
G 08	965	509	8.2157	0.9137		Mr 34	3489	1865	9.8472	0.9063
G 09	653	379	7.9035	0.9227		Mr 40	5218	2877	9.9948	0.8698
G 10	480	301	7.7245	0.9382		Mr 43	3356	1962	9.6097	0.8786
G 11	468	297	7.7563	0.9442		R 01	1738	843	8.7903	0.9044
G 12	251	169	6.9814	0.9433		R 02	2279	1179	9.1346	0.8953
G 13	460	253	7.4490	0.9331		R 03	1264	719	8.7035	0.9171
G 14	184	129	6.6629	0.9503		R 04	1284	729	8.7736	0.9226
G 15	593	378	8.0810	0.9438		R 05	1032	567	8.3954	0.9178
G 16	518	292	7.6923	0.9393		R 06	695	432	8.1436	0.9302
G 17	225	124	6.5269	0.9386		Rt 01	968	223	6.2661	0.8033
H 01	2044	1079	8.8380	0.8772		Rt 02	845	214	6.3747	0.8234
H 02	1288	789	8.6954	0.9035		Rt 03	892	207	6.5420	0.8503
H 03	403	291	7.5293	0.9199		Rt 04	625	181	6.3644	0.8486
H 04	936	609	8.4426	0.9127		Rt 05	1059	197	6.5085	0.8539
H 05	413	290	7.6043	0.9296		Ru 01	2595	1240	9.1104	0.8866
Hw 01	282	104	6.0083	0.8967		Ru 02	17205	6073	10.5714	0.8411
Hw 02	1829	257	6.5548	0.8188		Ru 03	3853	1792	9.5531	0.8839
Hw 03	3507	521	7.0628	0.7826		Ru 04	753	422	8.0561	0.9237
Hw 04	7892	744	6.5388	0.6855		Ru 05	6025	2536	9.9181	0.8771
Hw 05	7620	680	7.0618	0.7505		Sl 01	756	457	8.1613	0.9236
Hw 06	12356	1039	7.2720	0.7257		Sl 02	1371	603	8.2723	0.8957
I 01	11760	3667	9.8671	0.8333		Sl 03	1966	907	8.7048	0.8860
I 02	6064	2203	9.4130	0.8476		Sl 04	3491	1102	8.2855	0.8199
I 03	854	483	8.1008	0.9086		Sl 05	5588	2223	9.6509	0.8680
I 04	3258	1237	8.9123	0.8676		Sm 01	1487	266	6.3481	0.7881
I 05	1129	512	8.0893	0.8988		Sm 02	1171	219	6.3632	0.8184
In 01	376	221	7.2975	0.9370		Sm 03	617	140	5.9515	0.8348
In 02	373	209	7.2140	0.9360		Sm 04	736	153	5.9275	0.8168
In 03	347	194	7.1780	0.9445		Sm 05	447	124	5.8972	0.8480
In 04	343	213	7.4299	0.9606		T 01	1551	611	7.6919	0.8311
In 05	414	188	6.9893	0.9252		T 02	1827	720	7.8474	0.8268
Kn 01	3713	1664	9.7114	0.9076		T 03	2054	645	7.5103	0.8047
Kn 02	4508	1738	9.7285	0.9039						

A survey of mean entropies is presented in Table 8.2. However, if we relativize each entropy and take the means of relative entropies, the picture changes slightly. The strongly analytic Polynesian languages are still at the bottom of the scale where English moves, too, but the other languages cannot be evaluated on this scale with certainty. Especially the first place of Indonesian (0.9407) shows that averaged relative entropy is either no good estimator of analytism/synthetism or much more texts from each language are necessary to get a more precise result or, finally, relative entropy can be a property of texts of special sort and can be used as an individual characteristic. In order to make progress in this direction, one should concentrate on the study of one language and analyze a great number of different texts, even in historical perspective.

Table 8.2
Means of entropies and relative entropies
of ranking in 20 languages (ranking by mean *H*)

Language	mean H	mean H_{rel}
Marathi	10.1010	0.9199
Latin	9.8373	0.9270
Kannada	9.5958	0.9073
Russian	9.4418	0.8825
Italian	8.8765	0.8712
Romanian	8.6568	0.9146
Slovenian	8.6150	0.8786
English	8.4597	0.8170
Czech	8.2967	0.9075
Hungarian	8.2219	0.9086
Tagalog	7.6832	0.8209
Bulgarian	7.5498	0.9264
German	7.5297	0.9283
Indonesian	7.2217	0.9407
Maori	6.8232	0.8107
Lakota	6.8028	0.8734
Hawaiian	6.7498	0.7766
Rarotongan	6.4111	0.8359
Marquesan	6.4057	0.8170
Samoan	6.0975	0.8212

In studying frequencies of word forms, entropy is not a measure of uncertainty but a measure of non uniformity. It practically means that it can replace the slope

of the sequence: If the concentration of frequencies is strong, the entropy is small; if all frequencies are equal, the entropy is maximal.

Using this fact, we can try to set up a coordinate system <I,S> analogous to that of J.K. Ord (1972) who applied the first three moments and defined $I = m_2/m'_1$ and $S = m_3/m_2$ where m'_1 is the mean, m_2 is the variance and m_3 is the third central moment used to express the skewness of a distribution. We define the coordinates as follows:

$$(8.5) \quad I = \frac{m_2}{m'_1} = \frac{s^2}{\bar{x}}$$

which is identical with that of Ord, and

$$(8.6) \quad J = \frac{H}{s}$$

that is, J = *Entropy/standard deviation*. The values necessary for computing these coordinates and the coordinates themselves are presented in Table 8.3.

Table 8.3
The *I*,*J* values of 145 texts in 20 languages

Text	*N*	*V*	*H*	Mean	Variance	Stdev	*I*	*J*
B 01	761	400	7.8973	116.41	15154	4.46	130.18	1.77
B 02	352	201	7.0994	63.11	3893	3.33	61.69	2.13
B 03	515	285	7.5827	87.54	7705	3.87	88.02	1.96
B 04	483	286	7.598	91.56	8061	4.09	88.04	1.86
B 05	406	238	7.3055	75.32	5597	3.71	74.31	1.97
B 06	687	388	7.8501	117.41	14965	4.67	127.46	1.68
B 07	557	324	7.7944	102.85	10198	4.28	99.15	1.82
B 08	268	179	7.107	63.54	3218	3.47	50.65	2.05
B 09	550	313	7.6576	96.78	9575	4.17	98.94	1.84
B10	556	317	7.6055	97.78	9901	4.22	101.26	1.80
Cz 01	1044	638	8.6163	205.60	41492	6.30	201.81	1.37
Cz 02	984	543	8.3282	162.90	28729	5.40	176.36	1.54
Cz 03	2858	1274	8.9529	311.39	148584	7.21	477.16	1.24
Cz 04	522	323	7.877	108.88	10156	4.41	93.28	1.79
Cz 05	999	556	8.1959	164.71	30916	5.56	187.70	1.47
Cz 06	1612	840	8.6111	234.37	69239	6.55	295.43	1.31
Cz 07	2014	862	8.4876	208.12	65593	5.71	315.17	1.49

Cz 08	677	389	7.9987	122.71	14639	4.65	119.30	1.72
Cz 09	460	259	7.412	80.96	6404	3.73	79.10	1.99
Cz 10	1156	638	8.4876	188.21	40382	5.91	214.56	1.44
E 01	2330	939	8.5197	216.70	75324	5.69	347.60	1.50
E 02	2971	1017	8.3972	292.62	80587	5.21	275.40	1.61
E 03	3247	1001	8.2471	193.00	72945	4.74	377.95	1.74
E 04	4622	1232	8.4634	223.17	100219	4.66	449.07	1.82
E 05	4760	1495	8.7676	286.47	165012	5.89	576.02	1.49
E 06	4862	1176	8.2191	198.75	86393	4.22	434.68	1.95
E 07	5004	1597	8.8057	303.63	191290	6.18	630.01	1.42
E 08	5083	985	7.901	151.60	52689	3.22	347.55	2.45
E 09	5701	1574	8.6865	272.25	168275	5.43	618.09	1.60
E 10	6246	1333	8.3391	211.33	101773	4.04	481.58	2.06
E 11	8193	1669	8.5906	254.77	154593	4.34	606.79	1.98
E 12	9088	1825	8.5717	258.37	180773	4.46	699.67	1.92
E 13	11265	1659	8.4674	219.51	125227	3.33	570.48	2.54
G 01	1095	539	8.0326	141.82	26218	4.89	184.87	1.64
G 02	845	361	7.7006	96.28	10724	3.56	111.38	2.16
G 03	500	281	7.4369	85.53	7745	3.94	90.55	1.89
G 04	545	269	7.353	76.60	6533	3.46	85.29	2.13
G 05	559	332	7.7183	105.56	11028	4.44	104.47	1.74
G 06	545	326	7.7918	105.60	10458	4.38	99.03	1.78
G 07	263	169	6.9781	58.47	2830	3.28	48.40	2.13
G 08	965	509	8.2157	147.81	24715	5.06	167.21	1.62
G 09	653	379	7.9035	117.94	14283	4.68	121.10	1.69
G 10	480	301	7.7245	100.76	9085	4.35	90.16	1.78
G 11	468	297	7.7563	100.99	8779	4.33	86.93	1.79
G 12	251	169	6.9814	59.92	2915	3.41	48.65	2.05
G 13	460	253	7.449	78.82	5959	3.60	75.60	2.07
G 14	184	129	6.6629	47.55	1695	3.04	35.65	2.19
G 15	593	378	8.081	127.84	14406	4.93	112.69	1.64
G 16	518	292	7.6923	91.97	8014	3.93	87.14	1.96
G 17	225	124	6.5269	39.83	1385	2.48	34.78	2.63
H 01	2044	1079	8.838	304.74	114730	7.49	376.48	1.18
H 02	1288	789	8.6954	253.40	64108	7.05	252.99	1.23
H 03	403	291	7.5293	107.23	9019	4.73	84.11	1.59
H 04	936	609	8.4426	205.16	38918	6.45	189.70	1.31
H 05	413	290	7.6043	104.73	8871	4.63	84.70	1.64
Hw 01	282	104	6.0083	27.18	791	1.67	29.10	3.60

Hw 02	1829	257	6.5548	40.64	2929	1.27	72.07	5.16
Hw 03	3507	521	7.0628	69.94	11924	1.84	170.49	3.84
Hw 04	7892	744	6.5388	75.05	17627	1.49	234.87	4.39
Hw 05	7620	680	7.0618	68.74	14183	1.36	206.33	5.19
Hw 06	12356	1039	7.272	91.91	31025	1.58	337.56	4.60
I 01	11760	3667	9.8671	677.98	995548	9.20	1468.40	1.07
I 02	6064	2203	9.413	457.55	394764	8.07	862.78	1.17
I 03	854	483	8.1008	146.06	23255	5.22	159.22	1.55
I 04	3258	1237	8.9123	275.26	125667	6.21	456.54	1.44
I 05	1129	512	8.0893	134.05	23157	4.53	172.75	1.79
In 01	376	221	7.2975	71.50	4699	3.54	65.72	2.06
In 02	373	209	7.214	66.40	4060	3.30	61.14	2.19
In 03	347	194	7.178	62.78	3414	3.14	54.38	2.29
In 04	343	213	7.4299	74.83	4186	3.49	55.94	2.13
In 05	414	188	6.9893	53.37	2891	2.64	54.17	2.65
Kn 01	3713	1664	9.7114	432.48	242553	8.08	560.84	1.20
Kn 02	4508	1738	9.7285	413.84	241082	7.31	582.55	1.33
Kn 18	4483	1782	9.7515	429.97	259130	7.60	602.67	1.28
Kn 19	1787	833	8.9712	230.20	60340	5.81	262.12	1.54
Kn 20	4556	1755	9.6909	415.38	247018	7.36	594.68	1.32
Kn 21	1455	790	8.938	237.96	59195	6.38	248.76	1.40
Kn 22	4554	1794	9.6289	410.44	252940	7.45	616.27	1.29
Kn 23	4685	1738	9.6444	398.04	237040	7.11	595.52	1.36
Kn 30	4499	2005	10.0072	521.98	349426	8.81	669.42	1.14
Kn 31	4672	1920	9.8862	467.44	307916	8.12	658.73	1.22
Lk 01	345	174	6.7685	50.07	2763	2.83	55.18	2.39
Lk 02	1633	479	7.3035	89.02	16029	3.13	180.06	2.33
Lk 03	809	272	6.8508	57.74	5525	2.61	95.69	2.62
Lk 04	219	116	6.2882	35.39	1235	2.37	34.90	2.65
Lt 01	3311	2211	10.5032	771.11	502130	12.31	651.18	0.85
Lt 02	4010	2334	10.2814	716.64	552975	11.74	771.62	0.88
Lt 03	4931	2703	10.5934	803.93	711221	12.01	884.68	0.88
Lt 04	4285	1910	9.8252	484.42	323913	8.69	668.66	1.13
Lt 05	1354	909	9.3625	319.82	84291	7.89	263.56	1.19
Lt 06	829	609	8.4581	230.46	38237	6.79	165.92	1.25
M 01	2062	398	6.9856	63.92	8273	2.00	129.43	3.49
M 02	1175	277	6.7198	50.52	4538	1.97	89.83	3.41
M 03	1434	277	6.5851	46.22	4010	1.67	86.76	3.94
M 04	1289	326	6.6909	58.68	6583	2.26	112.18	2.96

M 05	3620	514	7.1346	69.43	11231	1.76	161.76	4.05
Mq 01	2330	289	6.6095	44.63	3634	1.25	81.43	5.29
Mq 02	457	150	6.1063	33.63	1613	1.88	47.96	3.25
Mq 03	1509	301	6.5012	50.66	5018	1.82	99.05	3.57
Mr 15	4693	1947	9.9538	477.82	318901	8.24	667.41	1.21
Mr 16	3642	1831	9.8062	515.11	311271	9.24	604.28	1.06
Mr 17	4170	1853	10.0913	485.80	296050	8.43	609.41	1.20
Mr 18	4062	1788	10.6433	454.41	279275	8.29	614.59	1.28
Mr 20	3943	1725	10.4632	444.83	255507	8.05	574.39	1.30
Mr 21	3846	1793	10.1882	485.31	284020	8.59	585.23	1.19
Mr 22	4099	1703	10.3521	411.51	379672	9.62	922.63	1.08
Mr 23	4142	1872	10.1542	490.93	307637	8.62	626.64	1.18
Mr 24	4255	1731	10.6589	430.34	246035	7.60	571.72	1.40
Mr 26	4146	2038	10.2964	559.20	384714	9.63	687.97	1.07
Mr 30	5504	2911	9.8764	838.79	816378	12.18	973.28	0.81
Mr 31	5105	2617	10.012	739.01	694115	11.28	939.25	0.89
Mr 32	5195	2382	9.9799	615.18	511833	9.93	832.01	1.01
Mr 33	4339	2217	9.7898	636.54	455576	10.25	715.71	0.96
Mr 34	3489	1865	9.8472	550.83	330358	9.73	599.75	1.01
Mr 40	5218	2877	9.9948	854.97	812869	12.48	950.76	0.80
Mr 43	3356	1962	9.6097	617.38	380198	10.64	615.82	0.90
R 01	1738	843	8.7903	228.99	65614	6.14	286.54	1.43
R 02	2279	1179	9.1346	328.59	135321	7.71	411.82	1.18
R 03	1264	719	8.7035	218.49	51552	6.39	235.95	1.36
R 04	1284	729	8.7736	222.11	52627	6.40	236.94	1.37
R 05	1032	567	8.3954	169.81	31234	5.50	183.93	1.53
R 06	695	432	8.1436	141.44	19066	5.24	134.80	1.55
Rt 01	968	223	6.2661	38.73	2888	1.73	74.57	3.62
Rt 02	845	214	6.3747	39.07	2794	1.82	71.51	3.50
Rt 03	892	207	6.542	40.76	2490	1.67	61.09	3.92
Rt 04	625	181	6.3644	37.52	2174	1.87	57.94	3.40
Rt 05	1059	197	6.5085	37.92	2026	1.38	53.43	4.72
Ru 01	2595	1240	9.1104	323.63	144401	7.46	446.19	1.22
Ru 02	17205	6073	10.5714	1215.70	2977793	13.16	2449.45	0.80
Ru 03	3853	1792	9.5531	454.98	299292	8.81	657.81	1.08
Ru 04	753	422	8.0561	129.43	17240	4.78	133.20	1.69
Ru 05	6025	2536	9.9181	598.93	566143	9.69	945.26	1.02
Sl 01	756	457	8.1613	146.80	21081	5.28	143.60	1.55
Sl 02	1371	603	8.2723	153.32	31702	4.81	206.77	1.72

Sl 03	1966	907	8.7048	235.20	74768	6.17	317.89	1.41
Sl 04	3491	1102	8.2855	213.74	89971	5.08	420.94	1.63
Sl 05	5588	2223	9.6509	502.76	421440	8.68	838.25	1.11
Sm 01	1487	267	6.3481	41.44	3657	1.57	88.25	4.04
Sm 02	1171	222	6.3632	38.36	2608	1.49	67.99	4.27
Sm 03	617	140	5.9515	26.46	1087	1.33	41.08	4.47
Sm 04	736	153	5.9275	27.39	1261	1.31	46.04	4.52
Sm 05	447	124	5.8972	25.91	979	1.48	37.78	3.98
T 01	1551	611	7.6919	133.62	32137	4.55	240.51	1.69
T 02	1827	720	7.8474	157.18	44773	4.95	284.85	1.59
T 03	2054	645	7.5103	119.45	30954	3.88	259.14	1.94

The graphical presentation in Figure 8.1 yields a very monolithic picture. All values lie on a convex decreasing curve, telling that word-form rank-frequencies have a very strictly regulated form. The theoretical background of this curve is not yet known. Since for fitting we used a simple function but for <*I*,*J*> a probability distribution argumentation, the only problem left for the future is to find a distribution whose <*I*,*J*> is similar to that in Figure 8.1.

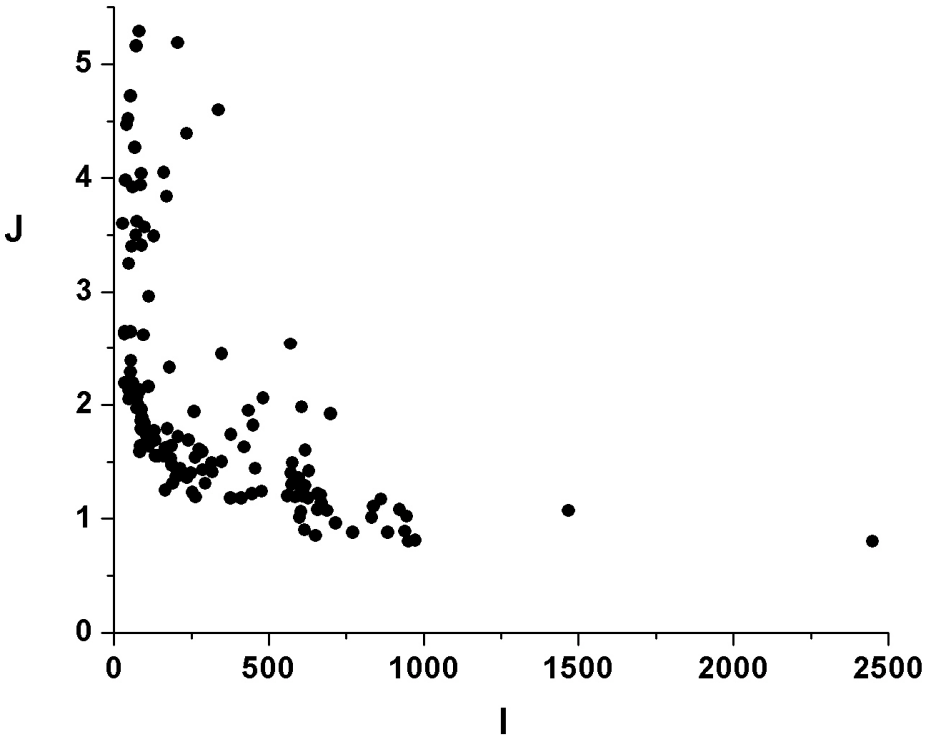


Figure 8.1. The <*I*,*J*> coordinate system for 145 texts

In order to show the individual languages in this coordinate system, we take averages and present the results in Table 8.4 and Figure 8.2. As can be seen, the upper part is covered by strongly analytic languages, the lower part by rather synthetic languages. The position of the more synthetic languages on the I-axis cannot still be interpreted, one needs more texts and more languages. Nevertheless, the numbers and the figure give a first orientation in language classification.

Table 8.4
The mean *I* and *J* values in individual languages

Language	*mean I*	*mean S*	Language	*mean I*	*mean J*
Bulgarian	91.97	1.89	Latin	567.60	1.03
Czech	215.99	1.54	Maori	115.99	3.57
English	493.45	1.85	Marathi	711.23	1.08
German	93.17	1.93	Marquesan	76.15	4.04
Hawaiian	175.07	4.46	Rarotongan	63.71	3.83
Hungarian	197.60	1.39	Romanian	248.33	1.40
Indonesian	58.27	2.26	Russian	926.38	1.16
Italian	623.94	1.40	Samoan	56.23	4.26
Kannada	539.16	1.31	Slovenian	385.49	1.48
Lakota	91.46	2.50	Tagalog	261.50	1.74

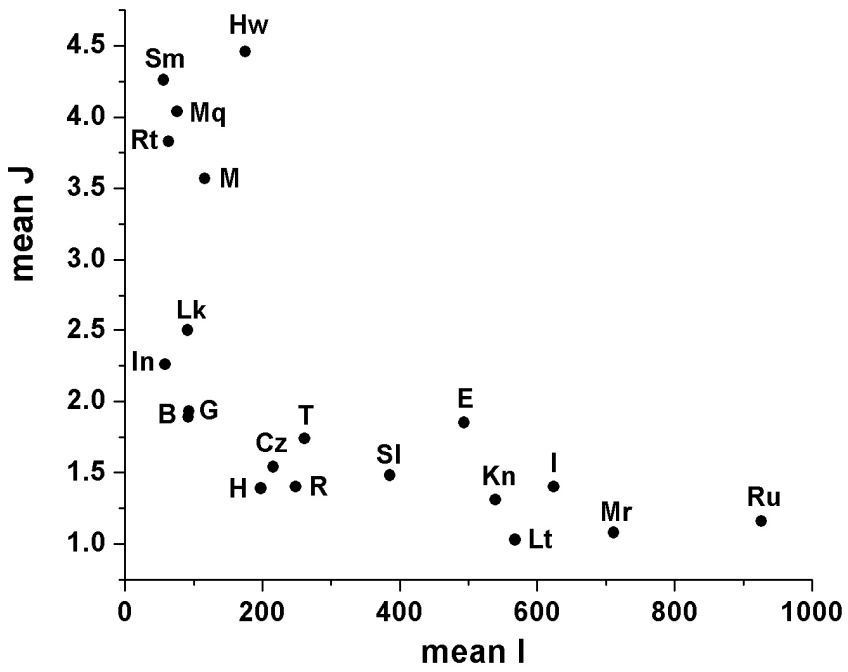


Figure 8.2. The new <I,J> scheme for 20 languages

The Repeat rate is defined as

$$(8.7)\quad R = \sum_{r=1}^{V} p_r^2$$

where p_r is estimated as above, $p_r = f_r/N$. Hence (8.7) can also be written as

$$(8.8)\quad R = \frac{1}{N^2}\sum_{r=1}^{V} f_r^2 \,.$$

The greater R, the smaller is the diversity of words. *R* is called also "measure of concentration" and can be interpreted in different ways (cf. Popescu et al. 2009). The asymptotic variance is given as

$$(8.9)\quad V(R) = \frac{4}{N}\left(\sum_{r=1}^{V} p_r^3 - R^2\right),$$

hence testing for difference is possible. *R* can be relativized in different ways, we use here simply

$$(8.10)\quad R_{rel} = \frac{1-R}{1-1/V}\,.$$

The values of *R* and R_{rel} are presented in Table 8.5.

Table 8.5
Repeat rate in 145 texts from 20 languages
(from Popescu et al. 2009, Table 9.23)

Text	*N*	*V*	*R*	R_{rel}		Text	*N*	*V*	*R*	R_{rel}
B 01	761	400	0.0092	0.9933		Kn 18	4485	1782	0.0035	0.9971
B 02	352	201	0.0012	1.0038		Kn 19	1787	833	0.0041	0.9971
B 03	515	285	0.0086	0.9949		Kn 20	4556	1755	0.0038	0.9968
B 04	483	286	0.0092	0.9943		Kn 21	1455	790	0.0047	0.9966
B 05	406	238	0.0112	0.993		Kn 22	4554	1794	0.0042	0.9964
B 06	687	388	0.0095	0.9931		Kn 23	4685	1738	0.0036	0.997
B 07	557	324	0.0076	0.9955		Kn 30	4499	2005	0.0032	0.9973
B 08	268	179	0.0105	0.9951		Kn 31	4672	1920	0.0028	0.9977
B 09	550	313	0.0093	0.9939		Lk 01	345	174	0.016	0.9897
B 10	556	317	0.0113	0.9918		Lk 02	1633	479	0.0181	0.984

Cz 01	1044	638	0.007	0.9946		Lk 03	809	272	0.0204	0.9832
Cz 02	984	543	0.0078	0.994		Lk 04	219	116	0.0214	0.9871
Cz 03	2858	1274	0.0086	0.9922		Lt 01	3311	2211	0.0027	0.9978
Cz 04	522	323	0.0076	0.9955		Lt 02	4010	2334	0.0038	0.9966
Cz 05	999	556	0.012	0.9898		Lt 03	4931	2703	0.0019	0.9985
Cz 06	1612	840	0.0101	0.9911		Lt 04	4285	1910	0.0034	0.9971
Cz 07	2014	862	0.0101	0.991		Lt 05	1354	909	0.003	0.9981
Cz 08	677	389	0.008	0.9946		Lt 06	829	609	0.0033	0.9983
Cz 09	460	259	0.0192	0.9846		M 01	2062	396	0.0209	0.9816
Cz 10	1156	638	0.0069	0.9947		M 02	1187	281	0.0241	0.9794
E 01	2330	939	0.0099	0.9912		M 03	1436	273	0.0252	0.9784
E 02	2971	1017	0.0098	0.9912		M 04	1409	302	0.0235	0.9797
E 03	3247	1001	0.0137	0.9873		M 05	3635	515	0.0182	0.9837
E 04	4622	1232	0.0139	0.9869		Mq 01	2330	289	0.0244	0.979
E 05	4760	1495	0.0103	0.9904		Mq 02	451	143	0.0288	0.978
E 06	4862	1176	0.0172	0.9836		Mq 03	1509	301	0.0379	0.9653
E 07	5004	1597	0.0096	0.991		Mr 15	4693	1947	0.0032	0.9973
E 08	5083	985	0.0192	0.9818		Mr 16	3642	1831	0.0024	0.9981
E 09	5701	1574	0.0102	0.9904		Mr 17	4170	1853	0.0024	0.9981
E 10	6246	1333	0.0159	0.9848		Mr 18	4062	1788	0.0034	0.9972
E 11	8193	1669	0.0129	0.9877		Mr 20	3943	1725	0.0026	0.998
E 12	9088	1825	0.012	0.9885		Mr 21	3846	1793	0.0022	0.9984
E 13	11625	1659	0.0119	0.9887		Mr 22	4099	1703	0.004	0.9966
G 01	1095	539	0.0117	0.9901		Mr 23	4142	1872	0.0026	0.9979
G 02	845	361	0.0108	0.9919		Mr 24	4255	1731	0.0028	0.9978
G 03	500	281	0.0122	0.9913		Mr 26	4146	2038	0.0025	0.998
G 04	545	269	0.0123	0.9914		Mr 30	5054	2911	0.0018	0.9985
G 05	559	332	0.0103	0.9927		Mr 31	5105	2617	0.002	0.9984
G 06	545	326	0.0087	0.9944		Mr 32	5195	2382	0.0024	0.998
G 07	263	169	0.0128	0.9931		Mr 33	4339	2217	0.0019	0.9986
G 08	965	509	0.0077	0.9943		Mr 34	3489	1865	0.0019	0.9986
G 09	653	379	0.0085	0.9941		Mr 40	5218	2877	0.0018	0.9985
G 10	480	301	0.0021	1.0012		Mr 43	3356	1962	0.0017	0.9988
G 11	468	297	0.0078	0.9956		R 01	1738	843	0.006	0.9952
G 12	251	169	0.0125	0.9934		R 02	2279	1179	0.0066	0.9942
G 13	460	253	0.0095	0.9944		R 03	1264	719	0.0065	0.9949
G 14	184	129	0.0144	0.9933		R 04	1284	729	0.0055	0.9959
G 15	593	378	0.0062	0.9964		R 05	1032	567	0.007	0.9948
G 16	518	292	0.0074	0.996		R 06	695	432	0.0072	0.9951

G 17	225	124	0.0153	0.9927		Rt 01	968	223	0.0338	0.9706
H 01	2044	1079	0.0155	0.9854		Rt 02	845	214	0.0256	0.979
H 02	1288	789	0.0133	0.988		Rt 03	892	207	0.0216	0.9831
H 03	403	291	0.0188	0.9846		Rt 04	625	181	0.0249	0.9805
H 04	936	609	0.0117	0.9899		Rt 05	1059	197	0.0202	0.9848
H 05	413	290	0.013	0.9904		Ru 01	2595	1240	0.0069	0.9939
Hw 01	282	104	0.0243	0.9852		Ru 02	17205	6073	0.0049	0.9953
Hw 02	1829	257	0.0206	0.9832		Ru 03	3853	1792	0.005	0.9956
Hw 03	3507	521	0.0211	0.9808		Ru 04	753	422	0.0079	0.9945
Hw 04	7892	744	0.0218	0.9795		Ru 05	6025	2536	0.0044	0.996
Hw 05	7620	680	0.0185	0.9829		Sl 01	756	457	0.0088	0.9934
Hw 06	12356	1039	0.0193	0.9816		Sl 02	1371	603	0.0078	0.9938
I 01	11760	3667	0.0055	0.9948		Sl 03	1966	907	0.0086	0.9925
I 02	6064	2203	0.0068	0.9937		Sl 04	3491	1102	0.0169	0.984
I 03	854	483	0.0106	0.9915		Sl 05	5588	2223	0.0054	0.995
I 04	3258	1237	0.0069	0.9939		Sm 01	1487	266	0.0309	0.9728
I 05	1129	512	0.0084	0.9935		Sm 02	1171	219	0.0273	0.9772
In 01	376	221	0.0101	0.9944		Sm 03	617	140	0.0282	0.9788
In 02	373	209	0.0108	0.994		Sm 04	736	153	0.034	0.9724
In 03	347	194	0.01	0.9951		Sm 05	447	124	0.0299	0.978
In 04	343	213	0.0077	0.997		T 01	1551	611	0.0165	0.9851
In 05	414	188	0.0115	0.9938		T 02	1827	720	0.0167	0.9847
Kn 01	3713	1664	0.0042	0.9964		T 03	2054	645	0.018	0.9835
Kn 02	4508	1738	0.0032	0.9974						

Again, taking only means for individual languages, we obtain the results in Table 8.6.

Table 8.6
Repeat rate means in individual languages
(ranking by decreasing mean R_{rel})

Language	*mean* R_{rel}	Language	*mean* R_{rel}	Language	*mean* R_{rel}
Marathi	0.9981	German	0.9939	Tagalog	0.9844
Latin	0.9977	Italian	0.9935	Hawaiian	0.9822
Kannada	0.9970	Czech	0.9922	Maori	0.9806
Russian	0.9950	Slovenian	0.9918	Rarotongan	0.9796
Romanian	0.9950	English	0.9880	Samoan	0.9758
Bulgarian	0.9949	Hungarian	0.9877	Marquesan	0.9741
Indonesian	0.9949	Lakota	0.9860		

The decrease of H_{rel} with decreasing synthetism is evident – at least visually – because the most synthetic languages are at the upper end and the most analytic ones at the lower end, the difference is not very conspicuous. Evidently, much more languages and much more texts in each of them are necessary to obtain a clear trend.

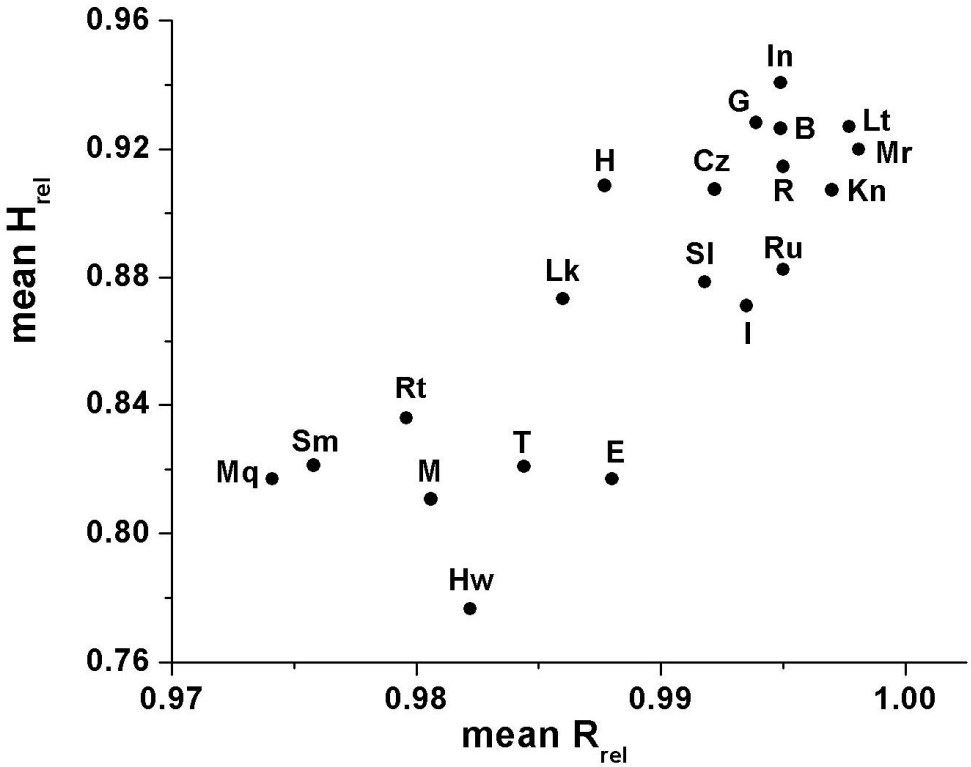


Figure 8.3. Mean H_{rel} agaist mean R_{rel}

The same holds for the relationship between H_{rel} and R_{rel}. A linear trend is evident, i.e. both tell the same story but some languages can be considered as outliers and further data must be won.

9. Nominal style

The concept of “nominal style“ designates a view from a special standpoint not necessarily adequate for every language. It is an attempt to characterize a text according to the weight of nouns in it. Different other “styles” are e.g. ornamental style taking into account the weight of adjectives, or active style taking into account that of verbs designating activity, etc. Usually, a special class of words is opposed to another class and their proportions are compared. Style is a kind of property and as such it is a matter of degree.

However, the concept of “nominal style” (or any other style) can be operationalized in many ways. It need not be simply the proportion of nouns in a text because in that case the given proportion should be compared with that in the population, but there are no populations in language (cf. Orlov, Boroda, Nadarejšvili 1982). It may be the comparison of the number of nouns and verbs in a text, as is usually made in German where nominal expressions are very frequently used in official language (e.g. *verbannen* vs. *in Verbannung schicken*; *beweisen* vs. *unter Beweis stellen*). It does not mean that a verb is replaced by a noun but that a noun is added. In this way the proportion of nouns in the text increases. If a language clearly distinguishes between nouns and verb, it is always possible to choose one of the above ways: taking the relative frequency of nouns in the given text or comparing their absolute frequency with that of some other word class. And since style is a choice among possibilities, properties of this type are characteristic of text, style, author, genre, etc. but not of language. A similar definition concerning *active* vs. *descriptive* style has been devised as early as in 1925 by A. Busemann.

The above situation can be captured either statically, counting the given words and weighting or comparing them, or dynamically, counting the position (time) of appearance of given words in text.

9.1. Static approach

One usually characterizes the situation using an indicator with the following properties: (a) It is simple, (b) it can be interpreted philologically, (c) it has a defined range, (d) it allows easy testing, i.e., its distribution or only some sampling properties are known.

Let us begin with a very simple indicator

$$(9.1) \quad I_1 = \frac{S}{S+V},$$

where S is the number of nouns and V that of verbs in a text. It is a simple proportion between <0,1>, S is distributed binomially, the variance is $Var(I_1) = I_1(1 - I_1)/(S + V)$ and its interpretation is as follows:

if $I_1 \approx 0.5$, then there is an equilibrium, i.e. to each noun there is a verb;

if $I_1 > 0.5$ significantly, then the text begins to be nominal, i.e. either many Vs are replaced by V+S, or verbal phrases contain additional nouns etc.;

if $I_1 < 0.5$ significantly, then the "verbality" grows, the nouns obtain more active predicates, the text gets more "active".

If the same indicator is defined using verbs and adjectives, one obtains a measure of descriptivity/activity (cf. Busemann 1925; Altmann 1978). Of course, the definition and the operationalization of the indicator depend on the aim of the analysis and on including or excluding copula, auxiliary verbs or modal verbs, too, etc.

Consider, e.g., the first strophe of Goethe´s "Erlkönig" in which there are 5 V (including all verbs) and 6 S. The above indicator yields $I_1 = 6/(6+5) = 0.5455$, i.e. there is an approximate nominal-verbal equilibrium and the result can be used for characterization. However, automatically the question arises how many nouns must accompany 5 verbs in order to consider the style significantly nominal. The problem can be solved very simply because the situation is binomial. Let $S + V = n$ and $E(I_1) = p = 0.5$ under the H_0 hypothesis of equilibrium. Our hypothesis H_1: $p > 0.5$, i.e. we ask what is the probability of finding S or more nouns. To this end one computes

$$(9.2) \quad P(X \geq S) = \sum_{x=S}^{n} \binom{n}{x} 0.5^n \quad .$$

the cumulative probability of the binomial distribution $b(n,p)$.

In our case with $S = 6$, $V = 5$, $n = 11$ we obtain $P(X \geq 6) = 0.5$ which is a sign of equilibrium. If we set $S = 7$, $V = 5$, $n = 12$, we obtain $P = 0.3872$, etc. With $S = 13$, $V = 5$, $n = 18$ we obtain $P = 0.048$, a number which is smaller than 0.05, the usual significance level. Thus the above text is rather in equilibrium than being nominal. Though I_1 can be used always for characterization, the test for significance can be performed also without computing the cumulative probability of the binomial distribution which gets tiresome if n is great. Another simple possibility is using the chi-square criterion yielding an approximate result.

Let the expected number of S and V be $n/2$ under the H_0 hypothesis, i.e., in equilibrium there is the same number of S and of V. Then the chi-square criterion for divergence is (ignoring all other classes of words)

$$(9.3) \quad X^2 = \frac{(S - \frac{n}{2})^2}{\frac{n}{2}} + \frac{(V - \frac{n}{2})^2}{\frac{n}{2}}.$$

Reordering the formula and replacing $n = S+V$ we obtain

$$(9.4) \quad X^2 = \frac{(S - V)^2}{S + V},$$

representing the chi-square with 1 degree of freedom. A more complex derivation using the likelihood ratio can be found in Altmann (1978, 1988: 23-29). Consider again the above example using $S = 13$, $V = 5$, then $X^2 = (13\text{-}5)^2/(13+5) = 3.5556$. The probability of this chi-square is 0.0593, slightly above the 0.05 level (which is at 3.64) being a good approximation to the binomial probability. The chi-square test is two-sided.

Since the root of chi-square with 1 degree of freedom is a normal variable, the root of (9.4) can be written in the form

$$(9.5) \quad u = \left(\frac{2S}{n} - 1\right)\sqrt{n}$$

representing a normal variable. Inserting the values $S = 13$, $n = 18$, we obtain

$$u = [2(13)/18\text{-}1]\sqrt{18} = 1.8856$$

which is exactly the root of X^2. The associated probability of u for a two-sided test is 0.0594, i.e. identical with that of the chi-square, slightly greater than the binomial probability which is exact.

For the complete "Erlkönig" with $S = 53$, $V = 41$ we obtain $P = 0.1282$, $X^2 = 1.53$, $u = 1.24$, i.e. a nominal-verbal equilibrium.

Now, if we only measure the extent of nominality, (9.1) is sufficient. But if we speak about significant nominality or verbality we can make our decisions in the following way:

(a) If $S > V$ and $P(X \geq S) \leq 0.05$ or $X^2 \geq 3.64$ or $u \geq 1.96$, we consider the style as nominal;
(b) if $S < V$ and $P(X \leq S) \leq 0.05$ or $X^2 \geq 3.64$ or $u \leq -1.96$, we consider the style as verbal;
(c) in all the other cases we speak about nominal-verbal equilibrium.

Needless to say, the chi-square has its weak points when the S and V are either too small or too large. If one is interested in text classification, one can set up

intervals using the binomial distribution or still simpler, the quantiles of the normal distribution associated with certain probabilities, or even still simpler, determining some conventional intervals for the indicator I_1. Ziegler, Best and Altmann (2002) analyzed 126 German texts and obtained very mixed results from significant verbality up to significant nominality signalizing the existence of different styles seen from this point of view. A thorough analysis of many texts in different genres and languages would be useful.

9.2. Dynamic approach

The relation between nouns and verbs need not be stable in a text. This impression of stability may arise if we take into account only their final numbers but do not observe the sequence of verbs and nouns. The sequence may get different forms which may tell us the story of text changing from nominality to activity, e.g. fairy tales which begin with telling us who is who, what properties the persons have, where they live etc. Bute later on, the persons begin to do something, the text gets active, active verbs occur more frequently. A different situation can arise in stage plays or in short stories. Nevertheless, the image of a text can have several forms depending on the consideration of active verbs (with or without auxiliaries, modal verbs, participles, etc.) or all verbs. However, in both cases one can perform the computation as follows.

Let X be the order number of the verb in a text that contains only nouns and verbs (all other words are omitted), and Y the number of nouns occurring up to the given X. For example, in a text *S V S S V S V* there is exactly one noun before the first verb, i.e. $x = 1$, $y = 1$; there are 3 nouns before the second verb, i.e. $x = 2$, $y = 3$, and for $x = 3$, $y = 4$. In this way we obtain a table

x	y
1	1
2	3
3	4

For $x = y$, representing the bisector, we have a state of equilibrium, but if $x > y$ or $x < y$ there is an infinite number of possible sequences. Nevertheless, every text can be characterized on the basis of its Y-sequence. Let us first consider some examples from German taken from Ziegler, Best, Altmann (2002).

(I) In the letter of the mayor of Vienna to the judge of Pressburg/Bratislava from the year 1427 which can be considered official, we find the following sequence (see Table 9.1).

Table 9.1
Official letter in German from 1427

The xth verb	y = number of nouns up to the xth verb
1	13
2	13
3	13
4	13
5	13
6	17
7	17
8	20
9	20
(10)	28

Position (10) is an auxiliary position enabling us to take into account also those nouns which occurred after the last verb position. The static test yields $S = 28$, $V = 9$, $P = 0.0013$, $X^2 = 9.76$, $u = 3.12$, i.e., a significant nominality. If we plot the points in a coordinate system we obtain a result as shown in Figure 9.1.

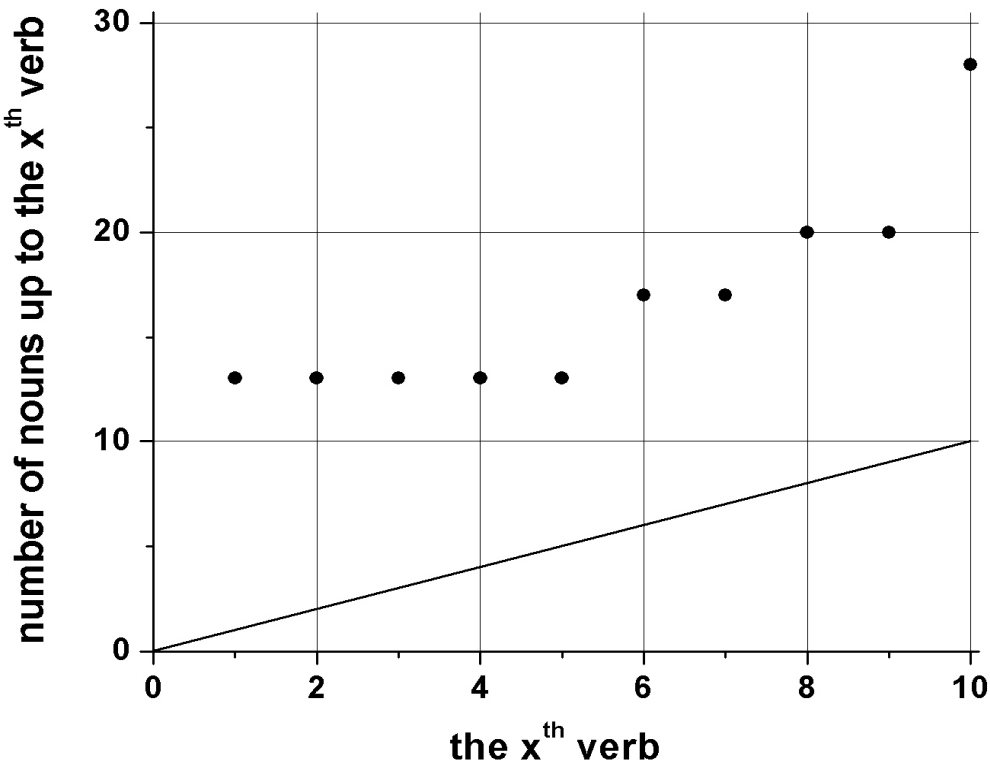


Figure 9.1. Nominality in a German Middle Ages official letter
(the straight line is $y = x$)

Without the last value $x = 10$, $y = 28$, the sequence could be considered linear, almost parallel to the bisector-equilibrium, but the last value $x = 10$, $y = 28$ renders the curve rather exponential: $y = 9.5806\exp(0.0940x)$ with $R^2 = 0.84$. The fitting of a straight line is not that efficient. However, if we consider a power function with an additive constant equal to the first y value, we obtain $y = 13 + 0.0029x^{3.6766}$ with $R^2 = 0.9407$. It is very probable, that all these curves have a form of a power function

(II) In Pestalozzi´s (1746-1827) short fable "Stoffels Brunnen" the following picture was found (cf. Table 9.2 and Figure 9.2)

Table 9.2
Nominality in Pestalozzi´s fable

x	*y*	*x*	*y*	*x*	*y*	*x*	*y*
1	2	10	9	19	17	28	23
2	2	11	10	20	17	29	23
3	4	12	11	21	20	30	23
4	4	13	12	22	20	31	23
5	7	14	12	23	21	32	23
6	7	15	14	24	21	33	23
7	7	16	15	25	21	34	23
8	8	17	15	26	21	35	23
9	8	18	15	27	23	36	23

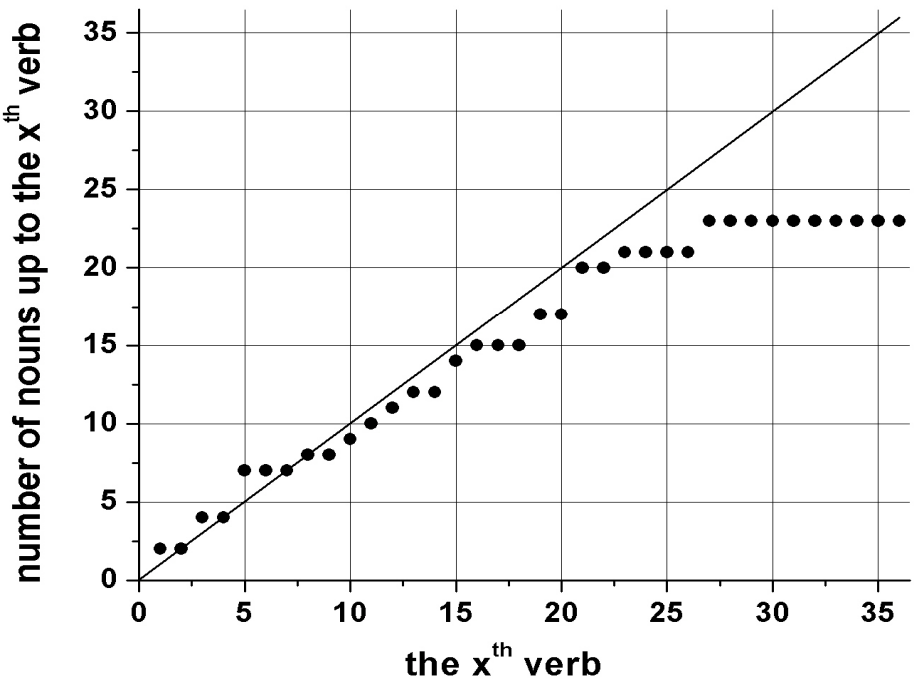


Figure 9.2. Pestalozzi´s fable "Stoffels Brunnen"

Here almost the whole sequence lies below the bisector and a straight line yields $R^2 = 0.9486$; however, a power function $y = 1.8708x^{0.7376}$ yields $R^2 = 0.9659$. Since $I_1 = 0.3898$ and the binomial test yields $P = 0.0587$, that is, it is at the boundary of significance; the verbality is not significant but the position of the two sequences in Figure 9.2 shows that there is at least a strong tendency towards verbality having a power form.

In this way any text can be analyzed in two ways, static and dynamic, and the number of possible aspects depends only on our concept formation. The study can be extended to typology, diachronic analysis, contrastive analysis etc. and can be used both in literary and in psycholinguistic studies. For the sake of comparability it is very important to tell how word classes are determined.

9.3. Prospects

The simple methods mentioned above furnish a number of research possibilities but each of them must be a priori grounded textologically. Here we shall merely give five hints.

One can begin with ancient philosophy and consider e.g. the categories of Aristotle: substance, quantity, quality, relatives, somewhere, sometime, being in a position, having, acting, being acted upon, and partition the texts in words-phrases-clauses-sentences expressing the categories. Either one characterizes the text in form of a vector containing categories as elements or one observes dynamically the change of the weight of individual categories. Needless to say, the philosophical literature proliferates on modifications of Aristotle´s views and texts could, perhaps, show how languages see the world. Aristotle´s classification is based on the properties of the Greek language.

Another possibility is to study the predicates. If the subject is noun or pronoun (which in some languages can be even elliptic), then verb and adjective are predicates of first order. The adverbs determining the adjectives and verbs are predicates of second order. The complements are of second order, too. Of course, there may be predicates of third order etc determining the predicates of second order. Thus a text can be rewritten as a sequence of ordinal numbers and this sequence can be studied both statically and dynamically.

Different types of grammar – if they are consequent – allow us to develop some measures of sentence properties. For example Tesnière´s dependence grammar allows us to evaluate the extent of dependence, centrality, depth and width of sentence (cf. Altmann, Lehfeldt 1973). The number of sentences having some degree of the qualities mentioned yields a frequency distribution, and the sequence of these indicators yields a dynamic picture of the text. Depth can be measured also using phrase structure grammar (cf. Yngve 1960) or its modern versions.

A stage play is a sequence of speech acts which can be either classified or quantified from a special point of view. It is to be expected that a comedy will have a different sequence of speech acts than a drama. Classical dramas have a very specific profile but comedies do not. Hence, the frequencies of speech acts and their sequences again yield a kind of text structure which can be captured quantitatively.

In psycholinguistics and language teaching, a number of different measures of text difficulty, dogmatism etc. have been developed. They have been used to characterize texts, persons etc. but usually testing or modeling the dynamics were omitted.

All this and many other vistas are possible and each of them would fill a book.

Appendix

Two centuries of German literature
253 texts of 26 German authors from Lessing to present days
(considered in tables 5.15, 7.8, and 7.11)

ID	author	text	period	mid year
Arnim 01	L.A. Arnim	Der tolle Invalide auf dem Fort Ratonneau	1781-1831	1806
Arnim 02	L.A. Arnim	Des ersten Bergmanns ewige Jugend	1781-1831	1806
Arnim 03	L.A. Arnim	Frau von Saverne	1781-1831	1806
Busch 01	W. Busch	Eduards Traum	1832-1908	1870
Chamisso 01	A. Chamisso	Peter Schlemihls wundersame Geschichte I	1781-1838	1810
Chamisso 02	A. Chamisso	Peter Schlemihls wundersame Geschichte II	1781-1838	1810
Chamisso 03	A. Chamisso	Peter Schlemihls wundersame Geschichte III	1781-1838	1810
Chamisso 04	A. Chamisso	Peter Schlemihls wundersame Geschichte IV	1781-1838	1810
Chamisso 05	A. Chamisso	Peter Schlemihls wundersame Geschichte V	1781-1838	1810
Chamisso 06	A. Chamisso	Peter Schlemihls wundersame Geschichte VI	1781-1838	1810
Chamisso 07	A. Chamisso	Peter Schlemihls wundersame Geschichte VII	1781-1838	1810
Chamisso 08	A. Chamisso	Peter Schlemihls wundersame Geschichte VIII	1781-1838	1810
Chamisso 09	A. Chamisso	Peter Schlemihls wundersame Geschichte IX	1781-1838	1810
Chamisso 10	A. Chamisso	Peter Schlemihls wundersame Geschichte X	1781-1838	1810
Chamisso 11	A. Chamisso	Peter Schlemihls wundersame Geschichte XI	1781-1838	1810
Droste 01	A. Droste-Huelshoff	Die Judenbuche	1797-1848	1823
Droste 02	A. Droste-Huelshoff	Der Tod des Erzbischofs Engelbert	1797-1848	1823
Droste 03	A. Droste-Huelshoff	Das Fegefeuer	1797-1848	1823
Droste 04	A. Droste-Huelshoff	Der Fundator	1797-1848	1823
Droste 05	A. Droste-Huelshoff	Die Schwestern	1797-1848	1823
Droste 08	A. Droste-Huelshoff	Der Geierpfiff	1797-1848	1823
Eichendorff 01	J.F. Eichendorff	Aus dem Leben eines Taugenichts 1	1788-1857	1823
Eichendorff 02	J.F. Eichendorff	Aus dem Leben eines Taugenichts 2	1788-1857	1823
Eichendorff 03	J.F. Eichendorff	Aus dem Leben eines Taugenichts 3	1788-1857	1823
Eichendorff 04	J.F. Eichendorff	Aus dem Leben eines Taugenichts 4	1788-1857	1823
Eichendorff 05	J.F. Eichendorff	Aus dem Leben eines Taugenichts 5	1788-1857	1823
Eichendorff 06	J.F. Eichendorff	Aus dem Leben eines Taugenichts 6	1788-1857	1823
Eichendorff 07	J.F. Eichendorff	Aus dem Leben eines Taugenichts 7	1788-1857	1823
Eichendorff 08	J.F. Eichendorff	Aus dem Leben eines Taugenichts 8	1788-1857	1823

Eichendorff 09	J.F. Eichendorff	Aus dem Leben eines Taugenichts 9	1788-1857	1823
Eichendorff 10	J.F. Eichendorff	Aus dem Leben eines Taugenichts 10	1788-1857	1823
Goethe 01	J.W. Goethe	Die neue Melusine	1749-1832	1791
Goethe 05	J.W. Goethe	Der Gott und die Bajadere	1749-1832	1791
Goethe 09	J.W. Goethe	Elegie 19	1749-1832	1791
Goethe 10	J.W. Goethe	Elegie 13	1749-1832	1791
Goethe 11	J.W. Goethe	Elegie 15	1749-1832	1791
Goethe 12	J.W. Goethe	Elegie 2	1749-1832	1791
Goethe 14	J.W. Goethe	Elegie 5	1749-1832	1791
Goethe 17	J.W. Goethe	Der Erlkönig	1749-1832	1791
Heine 01	H. Heine	Die Harzreise	1797-1856	1827
Heine 02	H. Heine	Die Heimkehr - Götterdämmerung	1797-1856	1827
Heine 03	H. Heine	Die Heimkehr - Die Wallfahrt nach Kevlaar	1797-1856	1827
Heine 04	H. Heine	Ideen. Das Buch Le Grand	1797-1856	1827
Heine 07	H. Heine	Belsazar	1797-1856	1827
Hoffmann 01	E.T.A. Hoffmann	Der Sandmann - Nathanael an Lothar	1776-1822	1799
Hoffmann 02	E.T.A. Hoffmann	Der Sandmann - Clara an Nathanael	1776-1822	1799
Hoffmann 03	E.T.A. Hoffmann	Der Sandmann - Nathanael an Lothar	1776-1822	1799
Immermann 01	K.L. Immermann	Der Karneval und die Somnabule	1796-1840	1818
Kafka 01	F. Kafka	In der Strafkolonie	1883-1924	1904
Kafka 02	F. Kafka	Ein Bericht für eine Akademie	1883-1924	1904
Kafka 03	F. Kafka	Betrachtung - Kinder auf der Landstraße	1883-1924	1904
Kafka 04	F. Kafka	Betrachtung - Entlarvung eines Bauernfängers	1883-1924	1904
Kafka 05	F. Kafka	Betrachtung - Der plötzliche Spaziergang	1883-1924	1904
Kafka 06	F. Kafka	Betrachtung - Entschlüsse	1883-1924	1904
Kafka 07	F. Kafka	Betrachtung - Der Ausflug ins Gebirge	1883-1924	1904
Kafka 08	F. Kafka	Betrachtung - Das Unglück des Junggesellen	1883-1924	1904
Kafka 09	F. Kafka	Betrachtung - Der Kaufmann	1883-1924	1904
Kafka 10	F. Kafka	Betrachtung - Zerstreutes Hinausschaun	1883-1924	1904
Kafka 11	F. Kafka	Betrachtung - Der Nachhauseweg	1883-1924	1904
Kafka 12	F. Kafka	Betrachtung - Die Vorüberlaufenden	1883-1924	1904
Kafka 13	F. Kafka	Betrachtung - Der Fahrgast	1883-1924	1904
Kafka 14	F. Kafka	Betrachtung - Kleider	1883-1924	1904
Kafka 15	F. Kafka	Betrachtung - Die Abweisung	1883-1924	1904
Kafka 16	F. Kafka	Betrachtung - Zum Nachdenken	1883-1924	1904
Kafka 17	F. Kafka	Betrachtung - Das Gassenfenster	1883-1924	1904
Kafka 18	F. Kafka	Betrachtung - Wunsch, Indianer zu werden	1883-1924	1904
Kafka 19	F. Kafka	Betrachtung - Die Bäume	1883-1924	1904
Kafka 20	F. Kafka	Betrachtung - Unglücklichsein	1883-1924	1904
Kafka 21	F. Kafka	Ein Brudermord	1883-1924	1904

Kafka 22	F. Kafka	Ein Landarzt	1883-1924	1904
Kafka 23	F. Kafka	Der Geier	1883-1924	1904
Kafka 24	F. Kafka	Vor dem Gesetz	1883-1924	1904
Kafka 25	F. Kafka	Ein Hungerkünstler	1883-1924	1904
Kafka 26	F. Kafka	Nachts	1883-1924	1904
Kafka 27	F. Kafka	Das Schweigen der Sirenen	1883-1924	1904
Kafka 28	F. Kafka	Die Sorge des Hausvaters	1883-1924	1904
Keller 01	G. Keller	Romeo und Julia auf dem Dorfe	1819-1890	1855
Keller 02	G. Keller	Vom Fichtenbaum	1819-1890	1855
Keller 03	G. Keller	Spiegel, das Kätzchen	1819-1890	1855
Keller 04	G. Keller	Das Tanzlegendchen	1819-1890	1855
Lessing 01	G.E. Lessing	Der Besitzer des Bogens	1729-1781	1755
Lessing 02	G.E. Lessing	Die Erscheinung	1729-1781	1755
Lessing 03	G.E. Lessing	Der Esel mit dem Löwen	1729-1781	1755
Lessing 04	G.E. Lessing	Der Fuchs	1729-1781	1755
Lessing 05	G.E. Lessing	Die Furien	1729-1781	1755
Lessing 06	G.E. Lessing	Jupiter und das Schaf	1729-1781	1755
Lessing 07	G.E. Lessing	Der Knabe und die Schlange	1729-1781	1755
Lessing 08	G.E. Lessing	Minerva	1729-1781	1755
Lessing 09	G.E. Lessing	Der Rangstreit der Tiere	1729-1781	1755
Lessing 10	G.E. Lessing	Zeus und das Pferd	1729-1781	1755
Löns 01	H. Löns	Der Werwolf - 1. Die Haidbauern	1866-1914	1890
Löns 02	H. Löns	Der Werwolf - 2. Die Mansfelder	1866-1914	1890
Löns 03	H. Löns	Der Werwolf - 3. Die Braunschweiger	1866-1914	1890
Löns 04	H. Löns	Der Werwolf - 4. Die Weimaraner	1866-1914	1890
Löns 05	H. Löns	Der Werwolf - 5. Die Marodebruede	1866-1914	1890
Löns 06	H. Löns	Der Werwolf - 6. Die Bruchbauern	1866-1914	1890
Löns 07	H. Löns	Der Werwolf - 7. Die Wehrwoelfe	1866-1914	1890
Löns 08	H. Löns	Der Werwolf - 8. Die Schnitter	1866-1914	1890
Löns 09	H. Löns	Der Werwolf - 9. Die Kirchenleute	1866-1914	1890
Löns 10	H. Löns	Der Werwolf - 10. Die Hochzeiter	1866-1914	1890
Löns 11	H. Löns	Der Werwolf - 11. Die Kaiserlichen	1866-1914	1890
Löns 12	H. Löns	Der Werwolf - 12. Die Schweden	1866-1914	1890
Löns 13	H. Löns	Der Werwolf - 13. Die Haidbauern	1866-1914	1890
Meyer 01	C.F. Meyer	Der Schuss von der Kanzel 1	1825-1898	1862
Meyer 02	C.F. Meyer	Der Schuss von der Kanzel 2	1825-1898	1862
Meyer 03	C.F. Meyer	Der Schuss von der Kanzel 3	1825-1898	1862
Meyer 04	C.F. Meyer	Der Schuss von der Kanzel 4	1825-1898	1862
Meyer 05	C.F. Meyer	Der Schuss von der Kanzel 5	1825-1898	1862
Meyer 06	C.F. Meyer	Der Schuss von der Kanzel 6	1825-1898	1862
Meyer 07	C.F. Meyer	Der Schuss von der Kanzel 7	1825-1898	1862
Meyer 08	C.F. Meyer	Der Schuss von der Kanzel 8	1825-1898	1862
Meyer 09	C.F. Meyer	Der Schuss von der Kanzel 9	1825-1898	1862
Meyer 10	C.F. Meyer	Der Schuss von der Kanzel 10	1825-1898	1862
Meyer 11	C.F. Meyer	Der Schuss von der Kanzel 11	1825-1898	1862
Novalis 01	H.O. Novalis	Heinrich von Ofterdingen - Die Erwartung 1	1772-1801	1787

Novalis 02	H.O. Novalis	Heinrich von Ofterdingen - Die Erwartung 2	1772-1801	1787
Novalis 03	H.O. Novalis	Heinrich von Ofterdingen - Die Erwartung 3	1772-1801	1787
Novalis 04	H.O. Novalis	Heinrich von Ofterdingen - Die Erwartung 4	1772-1801	1787
Novalis 05	H.O. Novalis	Heinrich von Ofterdingen - Die Erwartung 5	1772-1801	1787
Novalis 06	H.O. Novalis	Heinrich von Ofterdingen - Die Erwartung 6	1772-1801	1787
Novalis 07	H.O. Novalis	Heinrich von Ofterdingen - Die Erwartung 7	1772-1801	1787
Novalis 08	H.O. Novalis	Heinrich von Ofterdingen - Die Erwartung 8	1772-1801	1787
Novalis 09	H.O. Novalis	Heinrich von Ofterdingen - Die Erwartung 9	1772-1801	1787
Novalis 10	H.O. Novalis	Heinrich von Ofterdingen - Die Erfuellung	1772-1801	1787
Novalis 11	H.O. Novalis	Hyazinth und Rosenblütchen	1772-1801	1787
Novalis 12	H.O. Novalis	Neue Fragmente - Sophie	1772-1801	1787
Novalis 13	H.O. Novalis	Neue Fragmente - Traktat vom Licht	1772-1801	1787
Paul 01	J. Paul	1. Dr. Katzenbergers Badereise	1763-1825	1794
Paul 02	J. Paul	2. Reisezwecke	1763-1825	1794
Paul 03	J. Paul	3. Ein Reisegefaehrte	1763-1825	1794
Paul 04	J. Paul	4. Bona	1763-1825	1794
Paul 05	J. Paul	5. Herr von Niess	1763-1825	1794
Paul 06	J. Paul	6. Fortsetzung der Abreise	1763-1825	1794
Paul 07	J. Paul	7. Fortgesetzte Fortsetzung der Abreise	1763-1825	1794
Paul 08	J. Paul	8. Beschluss der Abreise	1763-1825	1794
Paul 09	J. Paul	9. Halbtagfahrt nach St. Wolfgang	1763-1825	1794
Paul 10	J. Paul	10. Mittags-Abenteuer	1763-1825	1794
Paul 11	J. Paul	11. Wagen-Sieste	1763-1825	1794
Paul 12	J. Paul	12. Die Avantuere	1763-1825	1794
Paul 13	J. Paul	13. Theodas ersten Tages Buch	1763-1825	1794
Paul 14	J. Paul	14. Missgeburten-Adel	1763-1825	1794
Paul 15	J. Paul	15. Hasenkrieg	1763-1825	1794
Paul 16	J. Paul	16. Ankunft-Sitzung	1763-1825	1794
Paul 17	J. Paul	I. Huldigungpredigt	1763-1825	1794
Paul 18	J. Paul	II. Ueber Hebels alemannische Gedichte	1763-1825	1794
Paul 19	J. Paul	III. Rat zu urdeutschen Taufnamen	1763-1825	1794
Paul 20	J. Paul	IIII. Dr. Fenks Leichenrede	1763-1825	1794
Paul 21	J. Paul	V. Ueber den Tod nach dem Tode	1763-1825	1794
Paul 22	J. Paul	17. Blosse Station	1763-1825	1794
Paul 23	J. Paul	18. Maennikes Seegefecht	1763-1825	1794
Paul 24	J. Paul	19. Mondbelustigungen	1763-1825	1794
Paul 25	J. Paul	20. Zweiten Tages Buch	1763-1825	1794

Paul 26	J. Paul	21. Hemmrad der Ankunft im Badeorte	1763-1825	1794
Paul 27	J. Paul	22. Niessiana	1763-1825	1794
Paul 28	J. Paul	23. Ein Brief	1763-1825	1794
Paul 29	J. Paul	24. Mittagtischreden	1763-1825	1794
Paul 30	J. Paul	25. Musikalisches Deklamatorium	1763-1825	1794
Paul 31	J. Paul	26. Neuer Gastrollenspieler	1763-1825	1794
Paul 32	J. Paul	27. Nachtrag	1763-1825	1794
Paul 33	J. Paul	28. Darum	1763-1825	1794
Paul 35	J. Paul	30. Tischgebet und Suppe	1763-1825	1794
Paul 36	J. Paul	31. Aufdeckung und Sternbedeckung	1763-1825	1794
Paul 37	J. Paul	32. Erkennszene	1763-1825	1794
Paul 38	J. Paul	33. Abendtisch-Reden ueber Schauspiele	1763-1825	1794
Paul 39	J. Paul	34. Brunnen-Beaengstigungen	1763-1825	1794
Paul 40	J. Paul	35. Theodas Brief an Bona	1763-1825	1794
Paul 41	J. Paul	36. Herzens-Interim	1763-1825	1794
Paul 42	J. Paul	37. Neue Mitarbeiter an allem	1763-1825	1794
Paul 43	J. Paul	I. Die Kunst, einzuschlafen	1763-1825	1794
Paul 44	J. Paul	II. Das Glueck	1763-1825	1794
Paul 45	J. Paul	III. Die Vernichtung	1763-1825	1794
Paul 46	J. Paul	38. Wie Katzenberger …	1763-1825	1794
Paul 47	J. Paul	39. Doktors Hoehlen-Besuch	1763-1825	1794
Paul 48	J. Paul	40. Theodas Hoehlen-Besuch	1763-1825	1794
Paul 49	J. Paul	41. Drei Abreisen	1763-1825	1794
Paul 50	J. Paul	42. Theodas kuerzeste Nacht der Reise	1763-1825	1794
Paul 51	J. Paul	43. Praeliminar-Frieden ...	1763-1825	1794
Paul 52	J. Paul	44. Die Stuben-Treffen	1763-1825	1794
Paul 53	J. Paul	45. Ende der Reisen und Noeten	1763-1825	1794
Paul 54	J. Paul	I. Wuensche fuer Luthers Denkmal	1763-1825	1794
Paul 55	J. Paul	II. Ueber Charlotte Corday	1763-1825	1794
Paul 56	J. Paul	III. Polymeter	1763-1825	1794
Pseudonym 01	Pseudonym (sloggi)	Eine kleine Geschichte mit der Zeit	2001	2001
Pseudonym 02	Pseudonym (sloggi)	Taumelnde Realitaet	2001	2001
Raabe 01	W. Raabe	Im Siegeskranze	1831-1910	1871
Raabe 02	W. Raabe	Eine Silvester-Stimmung	1831-1910	1871
Raabe 03	W. Raabe	Ein Besuch	1831-1910	1871
Raabe 04	W. Raabe	Deutscher Mondschein	1831-1910	1871
Raabe 05	W. Raabe	Theklas Erbschaft	1831-1910	1871
Rieder 01	E. Rieder	Liebe Mutter	2001	2001
Rieder 02	E. Rieder	Brief an einen Toten	2001	2001
Rückert 01	F. Rückert	Barbarossa	1788-1866	1827
Rückert 02	F. Rückert	Amor ein Besenbinder	1788-1866	1827
Rückert 03	F. Rückert	Der Frost	1788-1866	1827
Rückert 04	F. Rückert	Die goldne Hochzeit	1788-1866	1827
Rückert 05	F. Rückert	Erscheinung der Schnitterengel	1788-1866	1827
Schnitzler 01	A. Schnitzler	Der Sohn	1862-1931	1897

Schnitzler 02	A. Schnitzler	Albine	1862-1931	1897
Schnitzler 03	A. Schnitzler	Amerika	1862-1931	1897
Schnitzler 04	A. Schnitzler	Der Andere	1862-1931	1897
Schnitzler 05	A. Schnitzler	Die Braut	1862-1931	1897
Schnitzler 06	A. Schnitzler	Erbschaft	1862-1931	1897
Schnitzler 07	A. Schnitzler	Die Frau des Weisen	1862-1931	1897
Schnitzler 08	A. Schnitzler	Der Fürst ist im Hause	1862-1931	1897
Schnitzler 09	A. Schnitzler	Das Schicksal	1862-1931	1897
Schnitzler 10	A. Schnitzler	Welch eine Melodie	1862-1931	1897
Schnitzler 11	A. Schnitzler	Frühlingsnacht im Seziersaal	1862-1931	1897
Schnitzler 12	A. Schnitzler	Die Toten schweigen	1862-1931	1897
Schnitzler 13	A. Schnitzler	Er wartet auf den vazierenden Gott	1862-1931	1897
Schnitzler 14	A. Schnitzler	Mein Freund Ypsilon	1862-1931	1897
Sealsfield 01	C. Sealsfield	Das Cajuetenbuch - Die Praerie am Jacinto	1793-1864	1829
Sealsfield 02	C. Sealsfield	Das Cajuetenbuch 1	1793-1864	1829
Sealsfield 03	C. Sealsfield	Das Cajuetenbuch 2	1793-1864	1829
Sealsfield 04	C. Sealsfield	Das Cajuetenbuch 3	1793-1864	1829
Sealsfield 05	C. Sealsfield	Das Cajuetenbuch 4	1793-1864	1829
Sealsfield 06	C. Sealsfield	Das Cajuetenbuch 5	1793-1864	1829
Sealsfield 07	C. Sealsfield	Das Cajuetenbuch 6	1793-1864	1829
Sealsfield 08	C. Sealsfield	Das Cajuetenbuch 7	1793-1864	1829
Sealsfield 09	C. Sealsfield	Das Cajuetenbuch 8	1793-1864	1829
Sealsfield 10	C. Sealsfield	Das Cajuetenbuch 9	1793-1864	1829
Sealsfield 11	C. Sealsfield	Das Cajuetenbuch 10	1793-1864	1829
Sealsfield 12	C. Sealsfield	Das Cajuetenbuch 11	1793-1864	1829
Sealsfield 13	C. Sealsfield	Das Cajuetenbuch 12	1793-1864	1829
Sealsfield 14	C. Sealsfield	Das Cajuetenbuch 13	1793-1864	1829
Sealsfield 15	C. Sealsfield	Das Cajuetenbuch 14	1793-1864	1829
Sealsfield 16	C. Sealsfield	Das Cajuetenbuch 15	1793-1864	1829
Sealsfield 17	C. Sealsfield	Das Cajuetenbuch 16	1793-1864	1829
Sealsfield 18	C. Sealsfield	Das Cajuetenbuch - Der Fluch Kishogues	1793-1864	1829
Sealsfield 19	C. Sealsfield	Das Cajuetenbuch - Der Kapitaen	1793-1864	1829
Sealsfield 20	C. Sealsfield	Das Cajuetenbuch - Callao 1825	1793-1864	1829
Sealsfield 21	C. Sealsfield	Das Cajuetenbuch - Havanna 1816	1793-1864	1829
Sealsfield 22	C. Sealsfield	Das Cajuetenbuch - Sehr Seltsam!	1793-1864	1829
Sealsfield 23	C. Sealsfield	Das Cajuetenbuch - Ein Morgen im Paradiese	1793-1864	1829
Sealsfield 24	C. Sealsfield	Das Cajuetenbuch - Selige Stunden	1793-1864	1829
Sealsfield 25	C. Sealsfield	Das Cajuetenbuch - Das Diner	1793-1864	1829
Sealsfield 26	C. Sealsfield	Das Cajuetenbuch - Der Abend	1793-1864	1829
Sealsfield 27	C. Sealsfield	Das Cajuetenbuch - Die Fahrt und die Kajuete	1793-1864	1829
Sealsfield 28	C. Sealsfield	Das Cajuetenbuch - Das Paradies der Liebe	1793-1864	1829
Storm 01	T. Storm	Der Schimmelreiter	1817-1888	1853
Sudermann 01	H. Sudermann	Die Reise nach Tilsit	1857-1928	1893

Tucholsky 01	K. Tucholsky	Schloss Gripsholm 1	1890-1935	1913
Tucholsky 02	K. Tucholsky	Schloss Gripsholm 2	1890-1935	1913
Tucholsky 03	K. Tucholsky	Schloss Gripsholm 3	1890-1935	1913
Tucholsky 04	K. Tucholsky	Schloss Gripsholm 4	1890-1935	1913
Tucholsky 05	K. Tucholsky	Schloss Gripsholm 5	1890-1935	1913
Wedekind 01	F. Wedekind	Mine-Haha I	1864-1918	1891
Wedekind 02	F. Wedekind	Mine-Haha II	1864-1918	1891
Wedekind 03	F. Wedekind	Mine-Haha III	1864-1918	1891
Wedekind 04	F. Wedekind	Mine-Haha IV	1864-1918	1891
Wedekind 05	F. Wedekind	Rabbi Esra	1864-1918	1891
Wedekind 06	F. Wedekind	Frühlingsstürme	1864-1918	1891
Wedekind 07	F. Wedekind	Silvester	1864-1918	1891
Wedekind 08	F. Wedekind	Der Verführer	1864-1918	1891

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